KNOWLEDGE INSTITUTE OF TECHNOLOGY

Fluid Mechanics and Machinery

by
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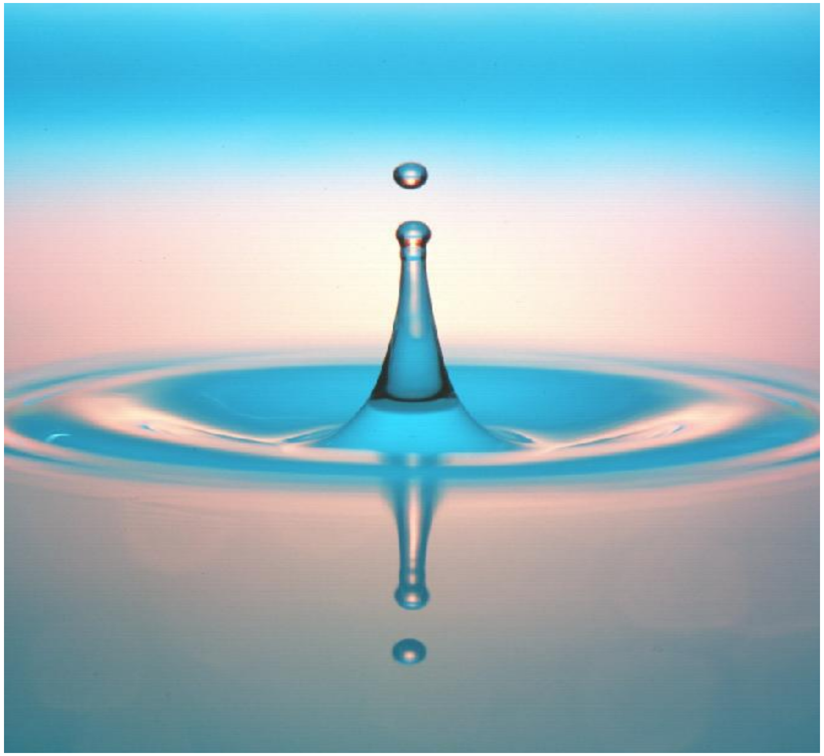
Topics going to Discuss

1. Introduction & Basic Concepts
2. Properties of Fluid
3. Fluid Statics
4. Fluid Kinematics
5. Energy Equations
6. Momentum Analysis of Fluid Flow
7. Dimensional Analysis
8. Flow in Pipes
9. Boundary Layer Theory

Fluid Mechanics is Beautiful

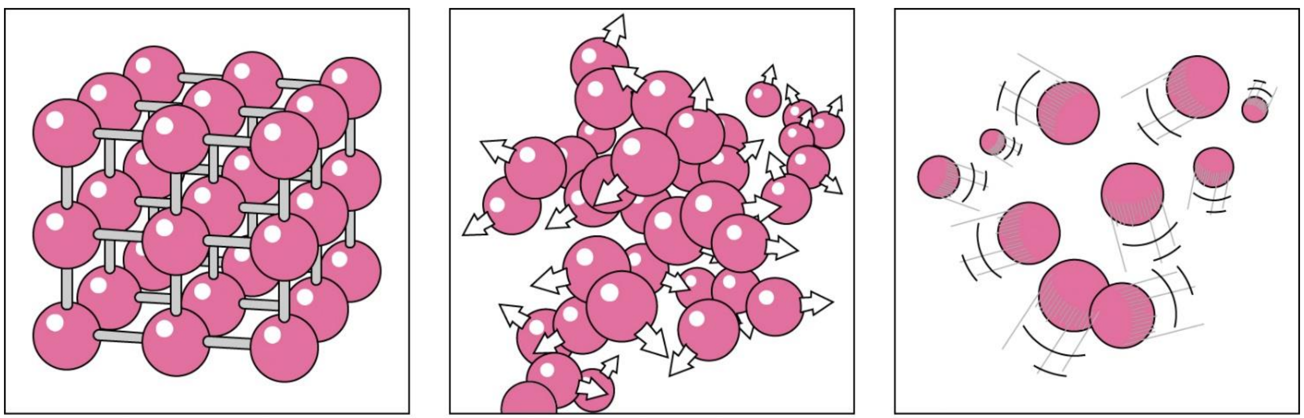


Introduction & Basic Concepts

The Concept of Solid, Liquid and Gas

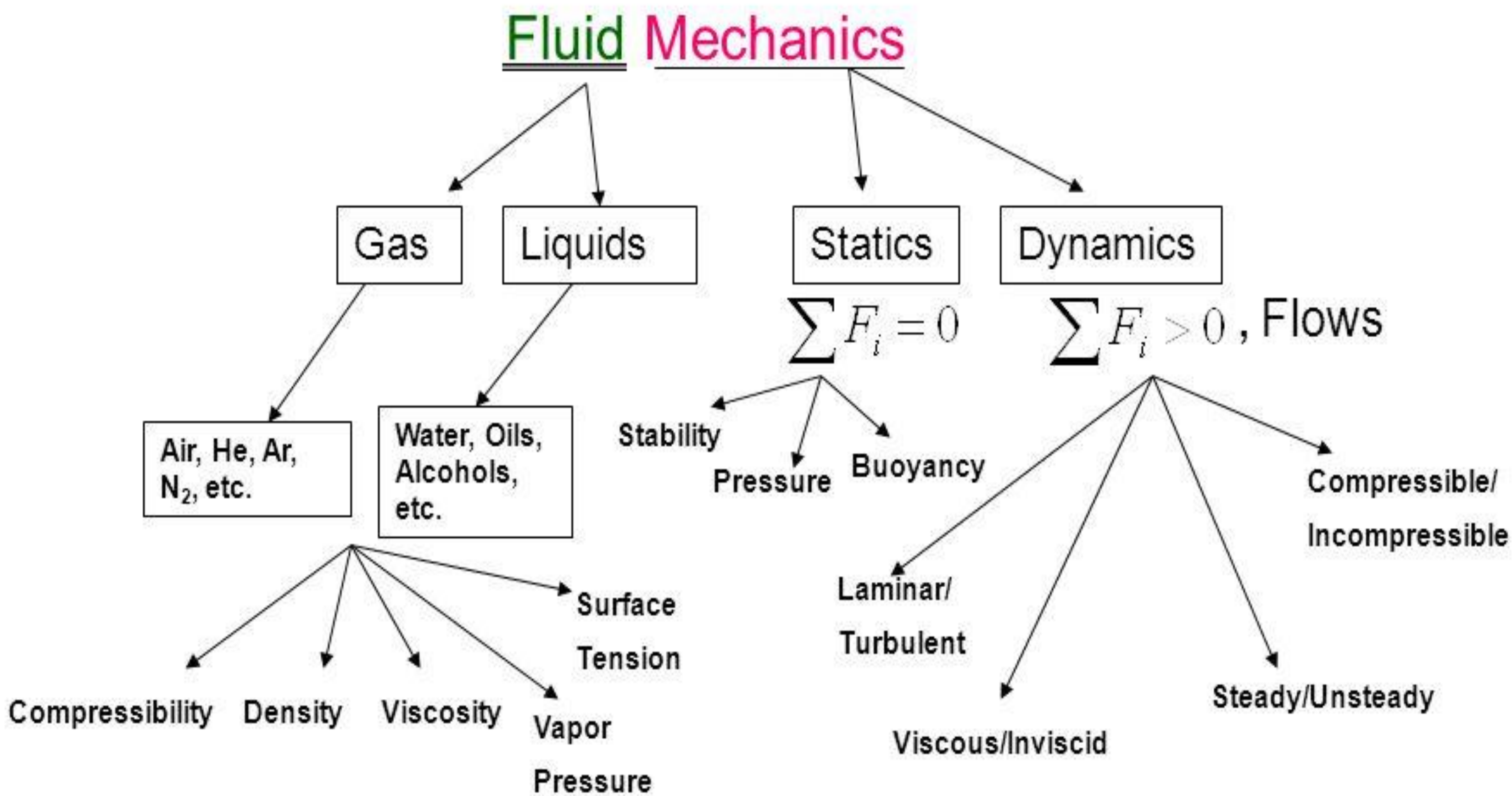
All matter consists of two states, solid and fluid

there are two classes of fluids, liquids and gases



(a) (b) (c)

solid liquid gas


Fluid Mechanics
Gas
Liquids
Statics
Dynamics
$\sum F_i = 0$
$\sum F_i > 0$, Flows
Air, He, Ar, N_2, etc.
Water, Oils, Alcohols, etc.
Stability
Pressure
Buoyancy
Compressible/ Incompressible
Laminar/ Turbulent
Steady/Unsteady
Viscous/Inviscid
Surface Tension
Compressibility
Density
Viscosity
Vapor Pressure

Basic concept

Equation of Motion

- Newton's Second Law

$$F = ma$$

or, equivalently,

- momentum principle:

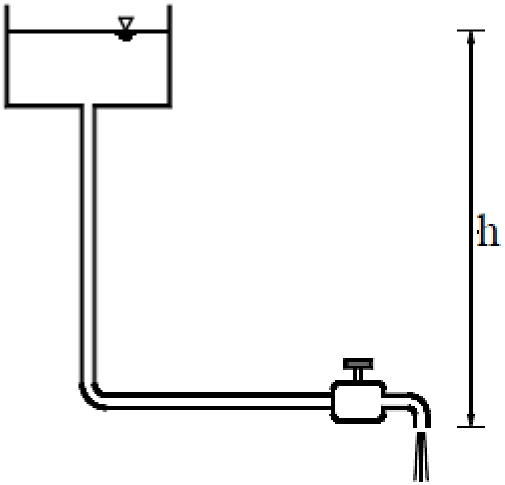
  force = rate of change of momentum

- mechanical energy principle:

  work done = Change of kinetic + potential energy

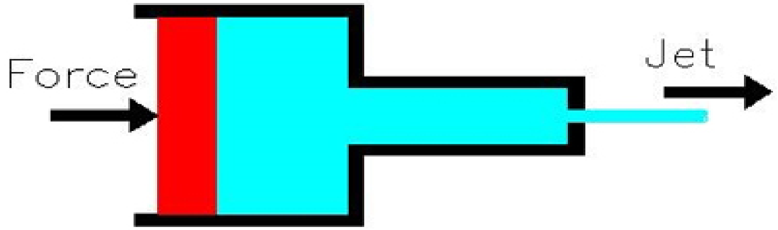
Power

rate of doing work (i.e. power) = force x velocity



$$H = \frac{potential\ energy}{mg} = energy\ per\ unit\ weight$$

$$Power = \frac{energy}{time} = \frac{mass}{time} \times gH$$



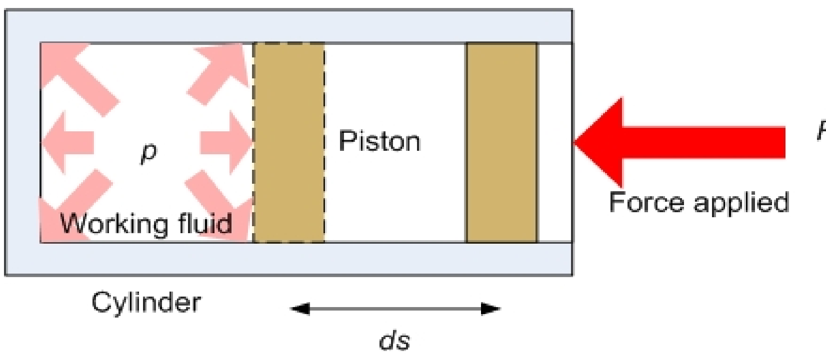
$$power = \rho QgH$$

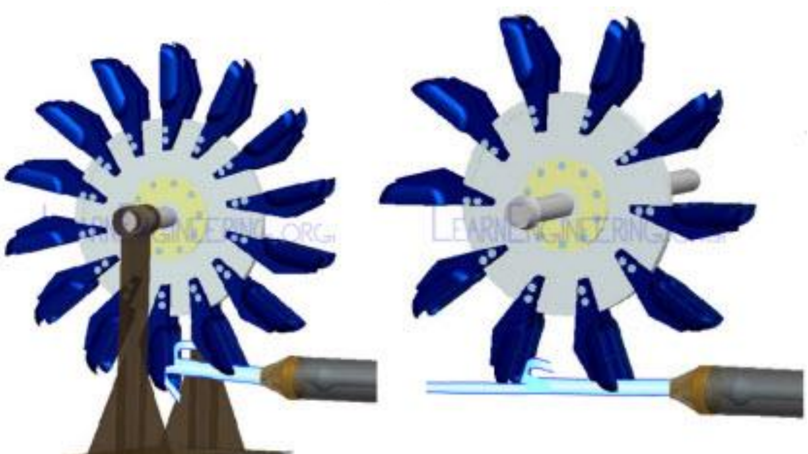
Work or Work done (Nm)

Work = force x parallel distance

$W = F \times d$

SI unit: **J - joules** (1 joules = 1Nm = 1kgm^2/s^2)

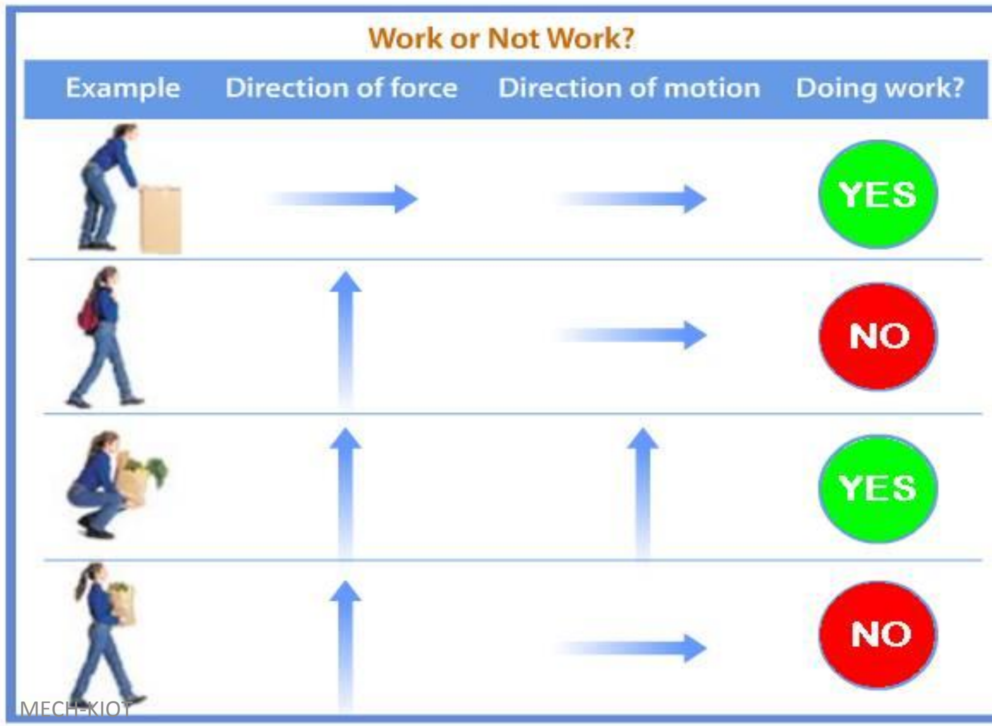




- A force that acts opposite to the direction of motion of an object does negative work
- No work (W=0) will be done when the displacement equals zero or when the force is perpendicular to the displacement.

A pushing force does ***no*** work if the wall does ***not*** move.

A pushing force ***does*** work if the wall moves even a little.



In each of the four situations shown, is work being done or not?

POWER

- Power is defined as **ability to do work**.
- SI Unit : Watt (W)
- **Formula:**

$$Power = \frac{Work}{time}$$

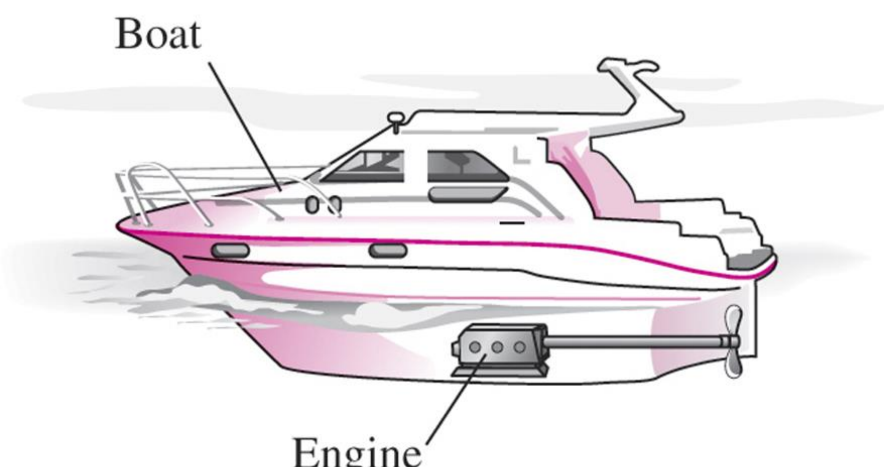
$$P = \frac{W}{t}$$

Joule

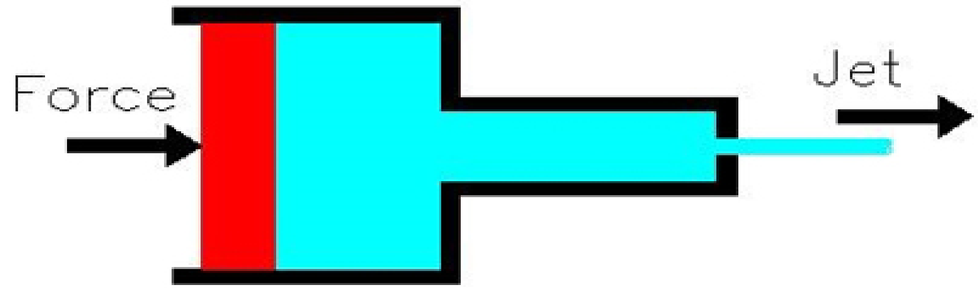
second

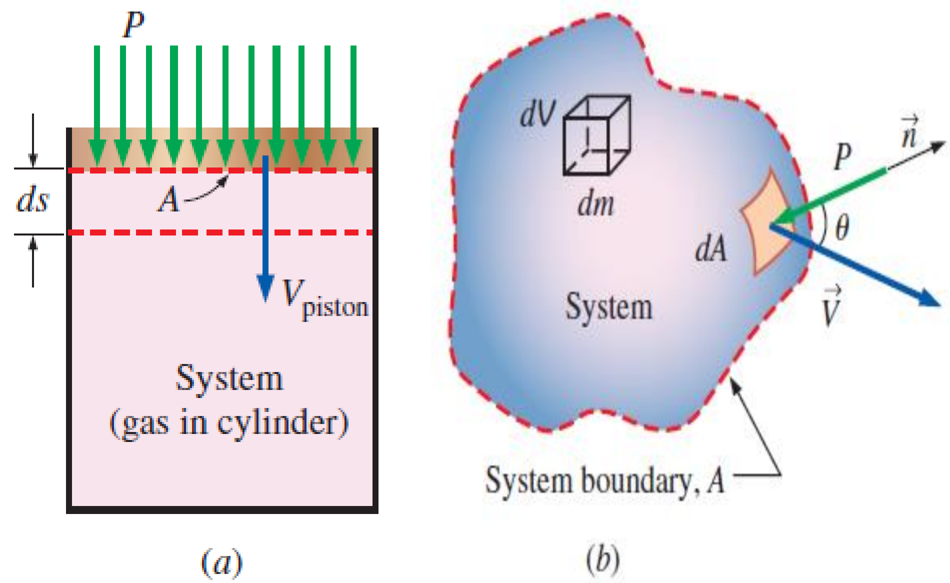
$$Power = \frac{Force \times displacement}{time}$$

$$Power = Force \times velocity$$



Energy transmission through rotating shafts is commonly encountered in practice.





The pressure force acting on (a) the moving boundary of a system in a piston-cylinder device, and (b) the differential surface area of a system of arbitrary shape.

Fluids

Fluids may be defined as substance which is capable of flowing. Fluids flow because of differences in pressure.

A fluid is a substance that flows under the action of shearing forces. If a fluid is at rest, we know that the forces on it are in balance.

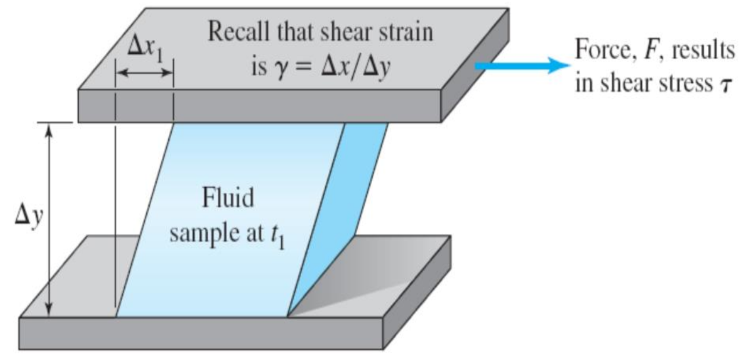
A gas is a fluid that is easily compressed. It fills any vessel in which it is contained.

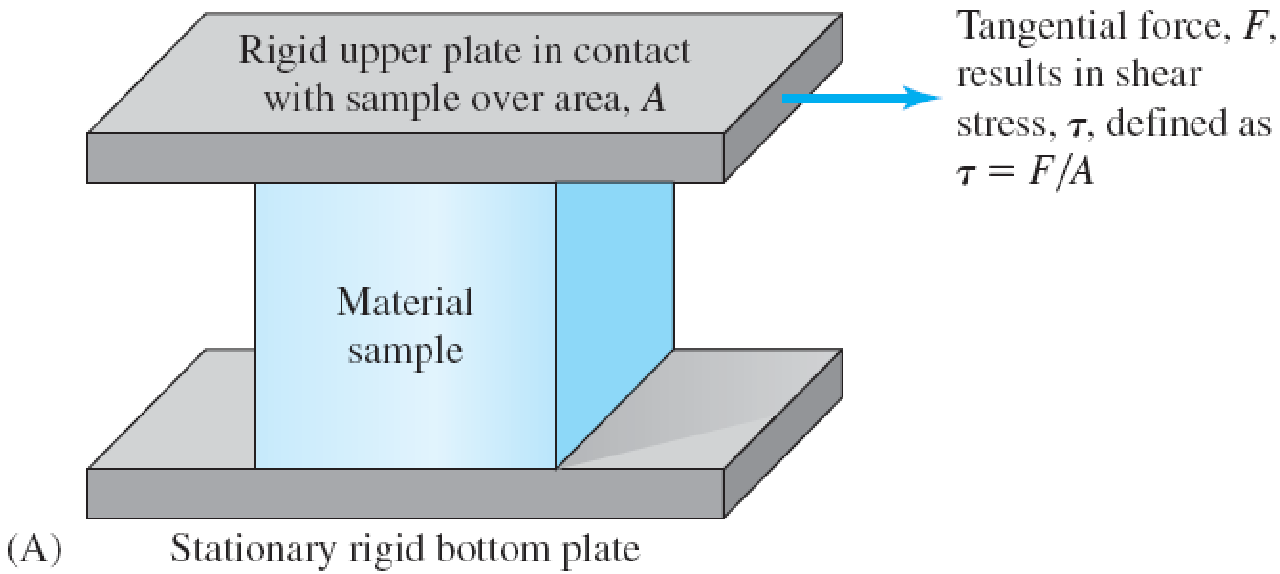
A liquid is a fluid which is hard to compress. A given mass of liquid will occupy a fixed volume, irrespective of the size of the container.

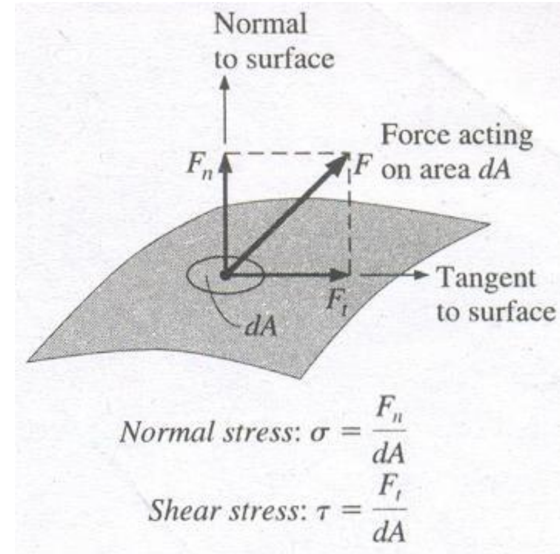
Fluid mechanics is the study of fluids

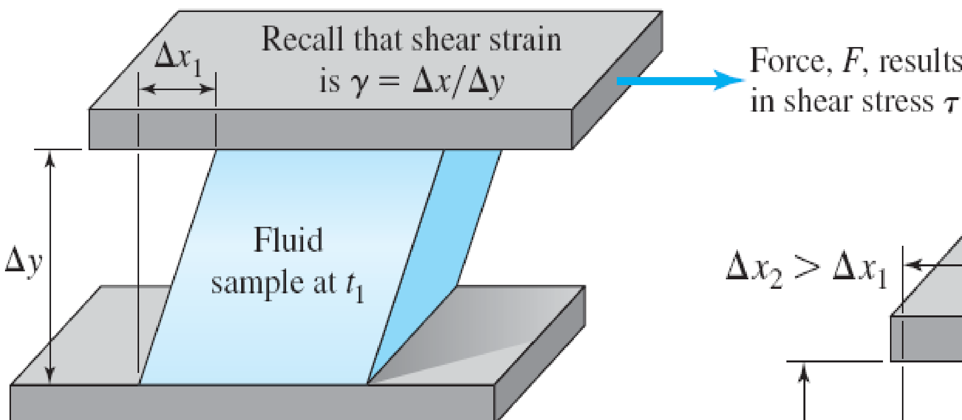
** at rest (fluid statics)*

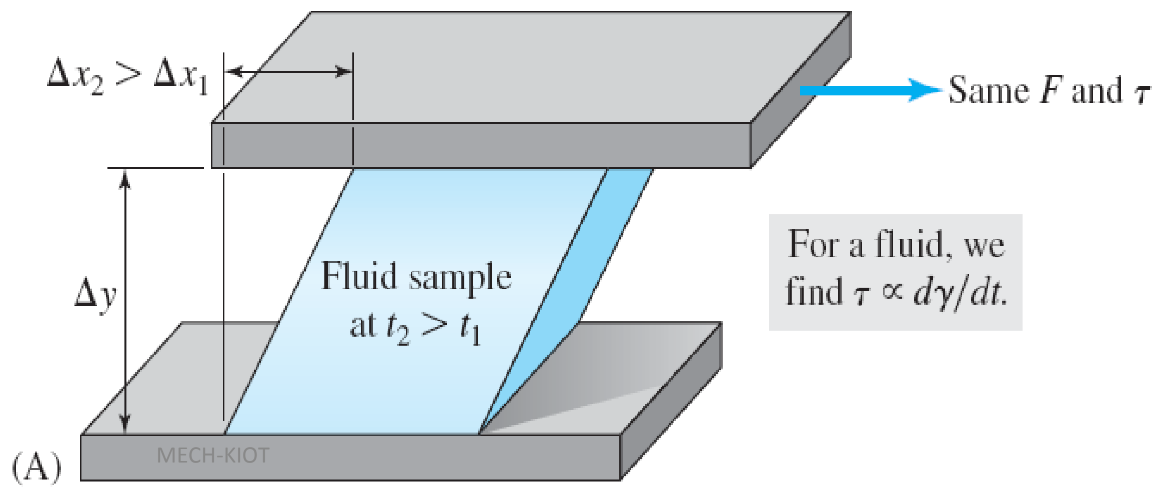
** in motion (fluid dynamics)*





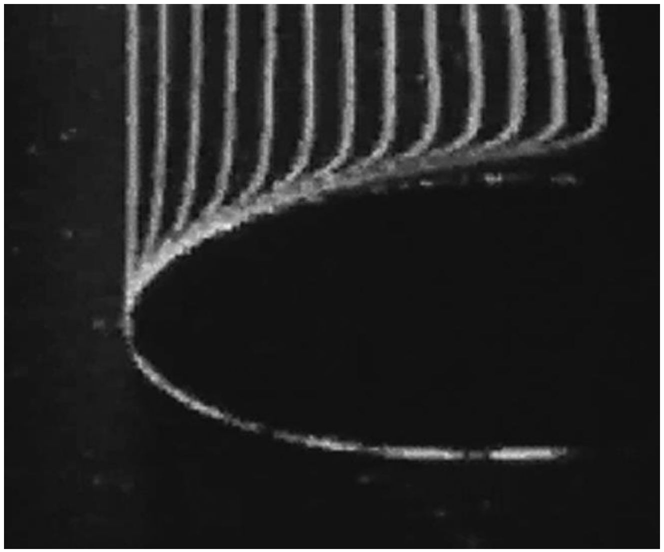


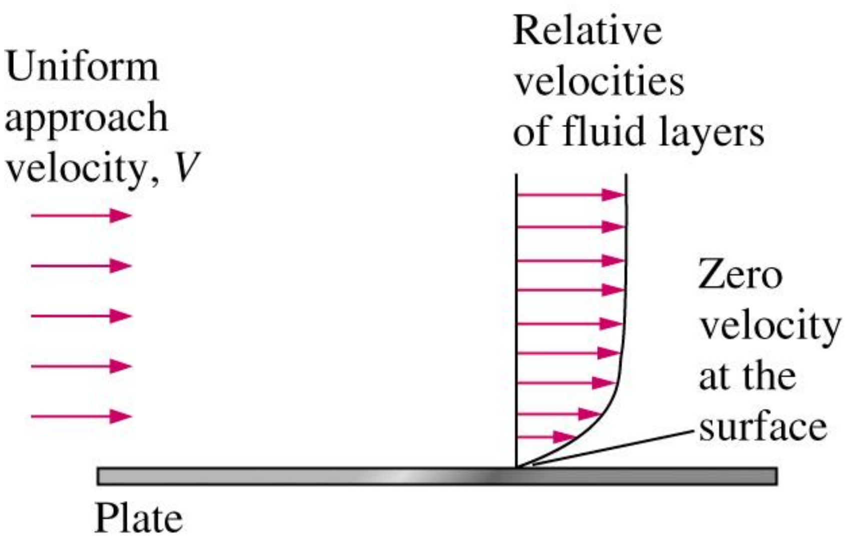




For a fluid, we find $\tau \propto d\gamma/dt$.

The No-Slip Condition



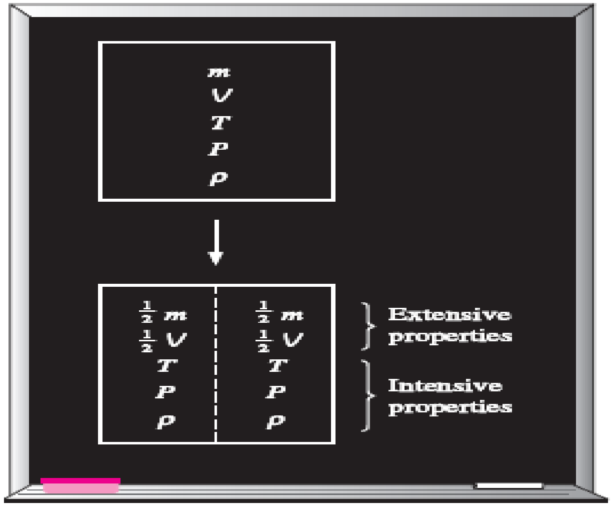


- No-slip condition: A fluid in direct contact with a solid "sticks" to the surface due to viscous effects
- Responsible for generation of wall shear stress τ_w,, surface drag $D= \int \tau_w \, dA$, and the development of the boundary layer
- Important boundary condition in formulating initial boundary value problem (IBVP) for analytical and computational fluid dynamics analysis

Properties of Fluids

Properties of a System

- Any characteristic of a system is called a property.
  - Familiar: pressure *P*, temperature *T*, volume *V*, and mass *m*.
  - Less familiar: viscosity(μ), thermal conductivity (k_T), modulus of elasticity (k), thermal expansion coefficient (α), Coefficient of volume expansion(β) vapor pressure, surface tension(σ).
- *Intensive* properties are independent of the mass of the system. Examples: temperature, pressure, and density.



Extensive properties are those whose value depends on the size of the system. Examples: Total mass, total volume, and total momentum.
Extensive properties per unit mass are called specific properties. Examples include specific volume *v = V/m* and specific total energy *e=E/m.*

Cont..

- In a given flow situation,

  the determination of the properties of the fluid either by experiment or theory as a function of position and time is considered to be the solution to the problem

- In almost all cases, the emphasis is on the space-time (x,y,z,t) distribution of the fluid properties

Density and Specific Gravity

- Density is defined as the mass per unit volume $\rho = m/V$. Density has units of kg/m^3
- Specific volume is defined as $v = 1/\rho = V/m$.
- For a gas, density depends on temperature and pressure.
- Specific gravity, or relative density is defined as the ratio of the density of a substance to the density of some standard substance at a specified temperature (usually water at 4°C), i.e., $SG=\rho/\rho_{H_2O}$. SG is a dimensionless quantity.
- The specific weight or weight density is defined as the weight per unit volume, i.e., $\rho_s = \rho g$ where g is the gravitational acceleration. g_s has units of N/m^3.

Vapor Pressure and Cavitation

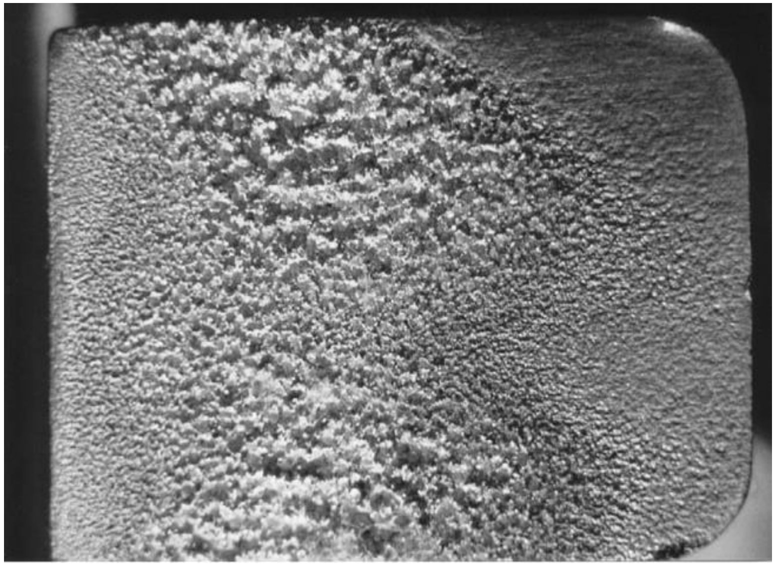
- Vapor Pressure P_v is defined as the pressure exerted by its vapor in phase equilibrium with its liquid at a given temperature

If P drops below P_v, liquid is locally vaporized, creating cavities of vapor.

Vapor cavities collapse when local P rises above P_v.

Collapse of cavities is a violent process which can damage machinery.

Cavitation is noisy, and can cause structural vibrations.



Cavitation Number

$$\mathrm{Ca} = \frac{p_a - p_v}{\frac{1}{2}\rho V^2}$$

Cavitation must avoided in flow systems since it reduces performance, generates annoying vibrations and noise and causes damage to equipment .

The large number of bubbles collapsing near the solid surface over a long period of time may cause erosion, surface pitting, fatigue failure and the destruction of the components or machinery.

The presence of cavitation in a flow system can be sensed by its characteristic tumbling sound

Energy and Specific Heats

- Total energy E is comprised of numerous forms: thermal, mechanical, kinetic, potential, electrical, magnetic, chemical, and nuclear.
- Units of energy are joule (J) or British thermal unit (BTU).
- Microscopic energy
  - Internal energy u is for a non-flowing fluid and is due to molecular activity.
  - Enthalpy $h=u+Pv$ is for a flowing fluid and includes flow energy (Pv).
- Macroscopic energy
  - Kinetic energy $ke=V^2/2$
  - Potential energy $pe=gz$
- In the absence of electrical, magnetic, chemical, and nuclear energy, the total energy is $e_{flowing}=h+V^2/2+gz$.

How does fluid volume change with P and T?

Fluids expand as $T\uparrow$ or $P\downarrow$

Fluids contract as $T\downarrow$ or $P\uparrow$

- The amount of volume change is different for different fluids
- Need fluid properties that relate volume changes to changes in P and T.
  - Coefficient of compressibility $$\kappa = -v\left(\frac{\partial P}{\partial v}\right)_T = \rho\left(\frac{\partial P}{\partial \rho}\right)_T$$
  - Coefficient of volume expansion $$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$$
- Combined effects of P and T can be written as $$dv = \left(\frac{\partial v}{\partial T}\right)_P dT + \left(\frac{\partial v}{\partial P}\right)_T dP$$

Coefficient of Compressibility

- The fluids act like elastic solids with respect to pressure.
- It is also called as bulk modulus of compressibility or bulk modulus of elasticity

$$\kappa = -v\left(\frac{\partial P}{\partial v}\right)_T = \rho\left(\frac{\partial P}{\partial \rho}\right)_T$$

- A larger value of k indicates that a large change in pressure is required to cause very small change in volume and thus a fluid with a large k is essentially incompressible.
  - coefficient of compressibility of an ideal gas $k_{ideal\ gas} = P$ (Pa)

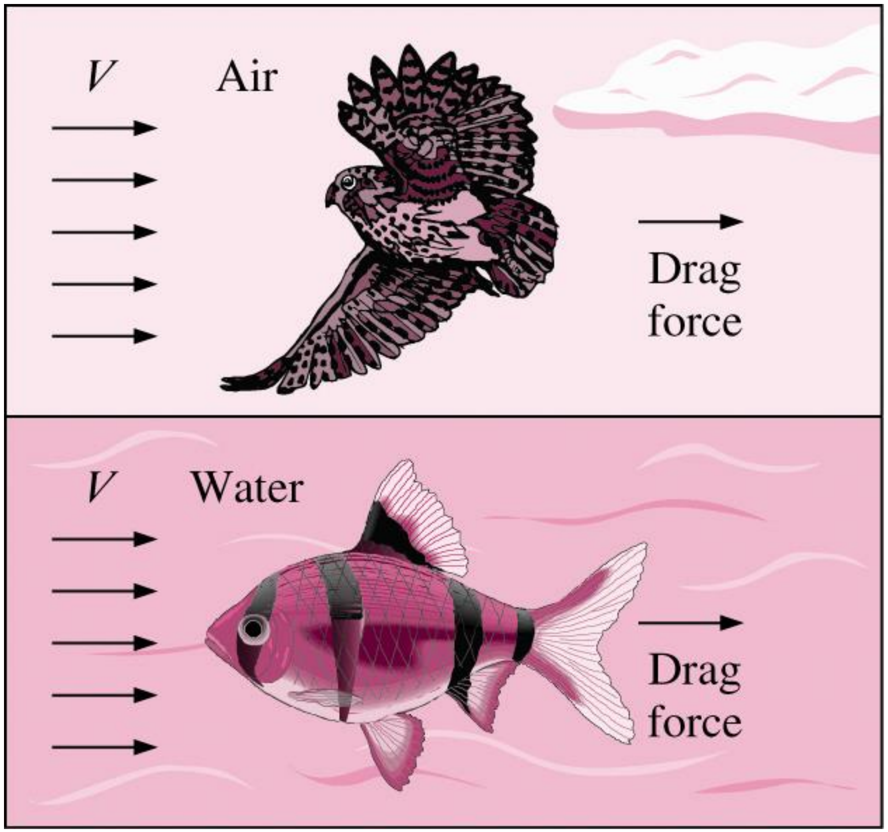
Isothermal Compressibility

- The inverse of the coefficient of compressibility is called the isothermal compressibility

$$\alpha = \frac{1}{k} = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_T = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial P}\right)$$

- The isothermal compressibility of a fluid represents the fractional change in volume or density corresponding to a unit change in pressure

Viscosity

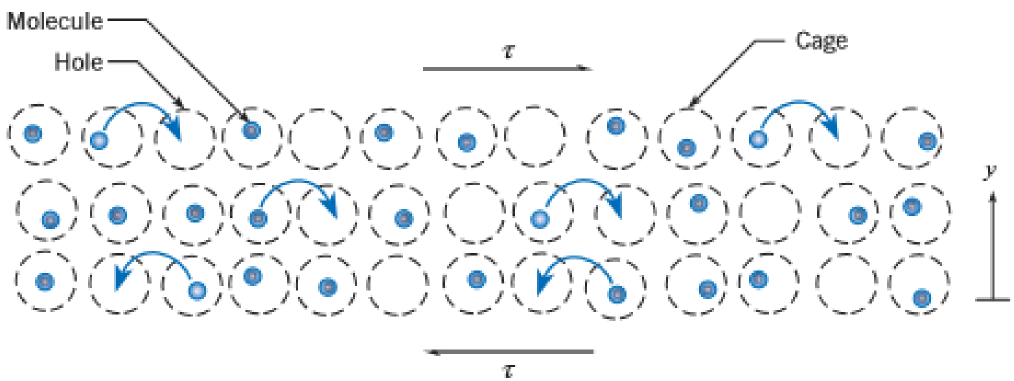


- Viscosity is a property that represents the internal resistance of a fluid to motion.

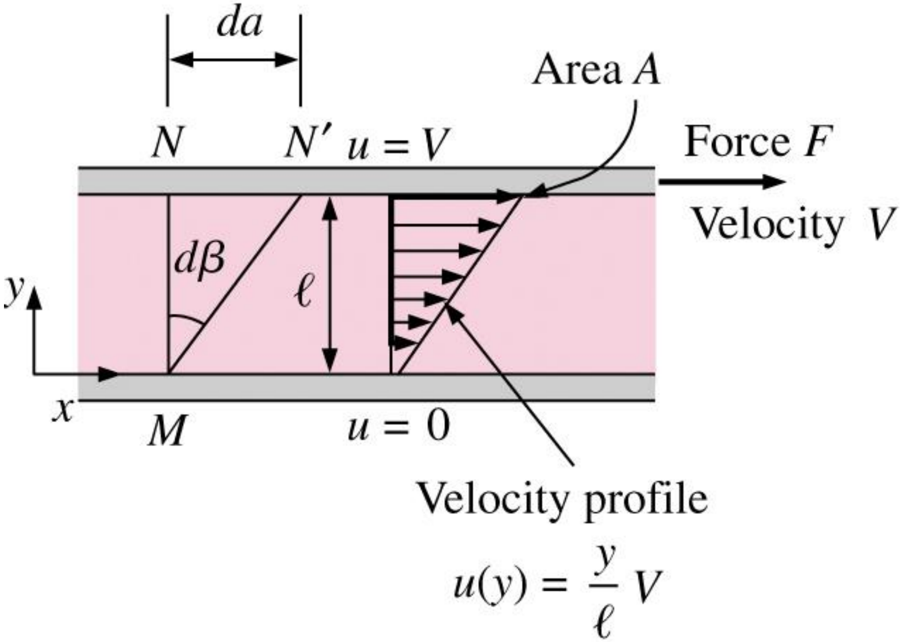
- The force a flowing fluid exerts on a body in the flow direction is called the drag force.

Viscosity

- Viscosity is the property of a fluid, due to cohesion and interaction between molecules, which offers resistance to sheer deformation. Different fluids deform at different rates under the same shear stress.



- Fluid with a high viscosity such as syrup, deforms more slowly than fluid with a low viscosity such as water.



- To obtain a relation for viscosity, consider a fluid layer between two very large parallel plates separated by a distance ℓ
- Definition of shear stress is $\tau = F/A$.
- Using the no-slip condition, $u(0) = 0$ and $u(\ell) = V$, the velocity profile and gradient are $u(y)= Vy/\ell$ and $du/dy=V/\ell$
- Shear stress for Newtonian fluid: $\tau = \mu\, du/dy$
- μ is the dynamic viscosity and has units of kg/m·s, Pa·s, or poise.

- Dynamic Viscosity

  $\mu = \tau (du/dy)$

- Dynamic viscosity is also called as absolute viscosity or coefficient of viscosity.

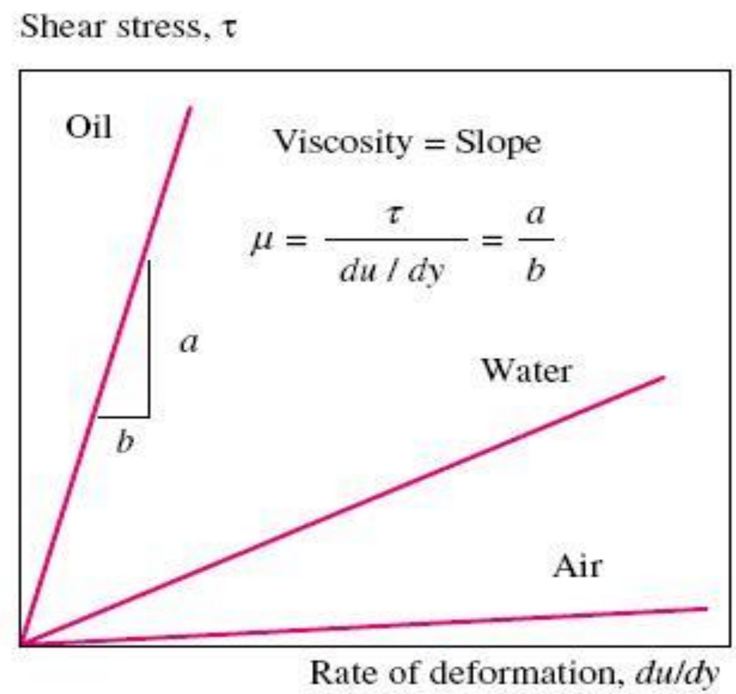
- Unit of dynamic viscosity kg/ms or Ns/m^2 or poise

  Poise = 0.1 Ns/m^2

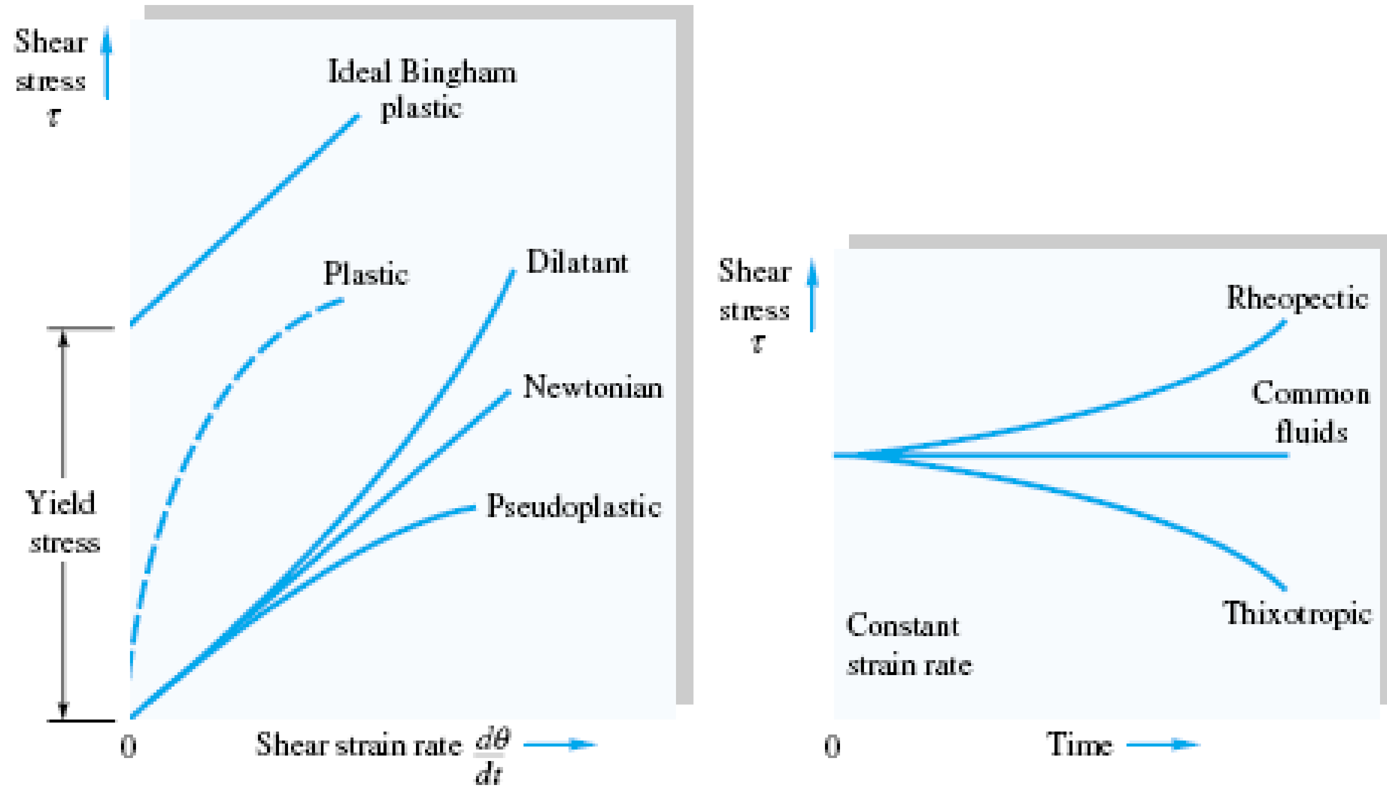
- Kinematic viscosity ν = dynamic viscosity / density

  Unit of Kinematic Viscosity is m^2/s

  1 Stoke = 1 cm^2/s = $10^{-4} m^2/s$



Types of fluids



Fluids which do not follow the linear law of viscosity are called nonnewtonian and also called rheological fluids .

Types of Non-Newtonian fluids:

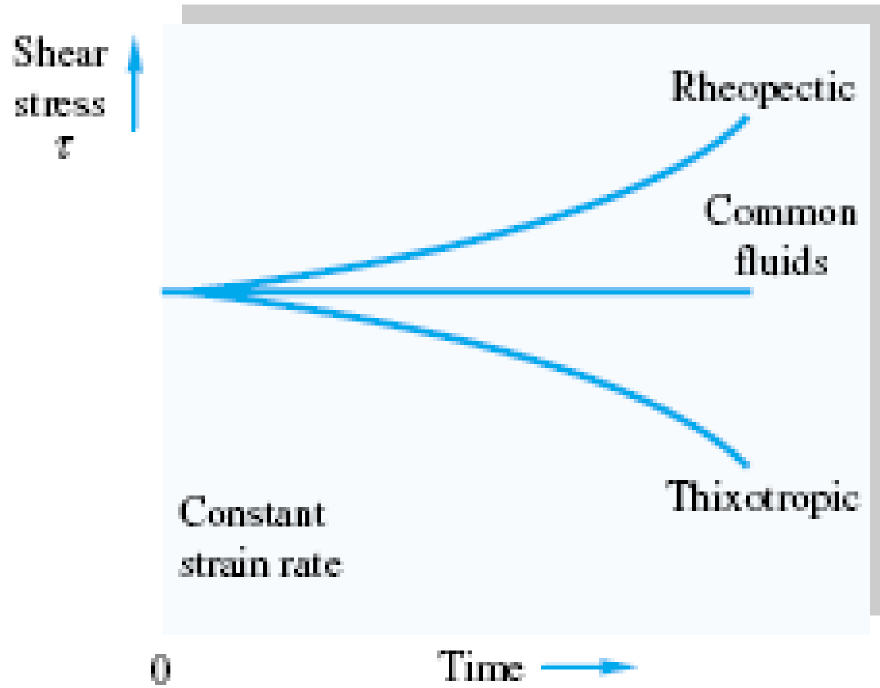
- Dilatant, or shear-thickening fluid increases resistance with increasing applied stress. Ex: Solutions with suspended starch and sand

- Pseudoplastic, or shear-thinning fluid decreases resistance with increasing stress. Ex: paints , polymer solutions

- If the thinning effect is very strong, as with the dashed-line curve, the fluid is termed plastic. The limiting case of a plastic substance is one which requires a finite yield stress before it begins to flow.

- Bingham plastic
  Flow behaviour after yield may also be nonlinear. An example of a yielding fluid is toothpaste, which will not flow out of the tube until a finite stress is applied by squeezing

- A further complication of nonnewtonian behavior is the transient effect shown in Fig below.



- Some fluids require a gradually increasing shear stress to maintain a constant strain rate and are called rheopectic.

- The opposite case of a fluid which thins out with time and requires decreasing stress is termed thixotropic.

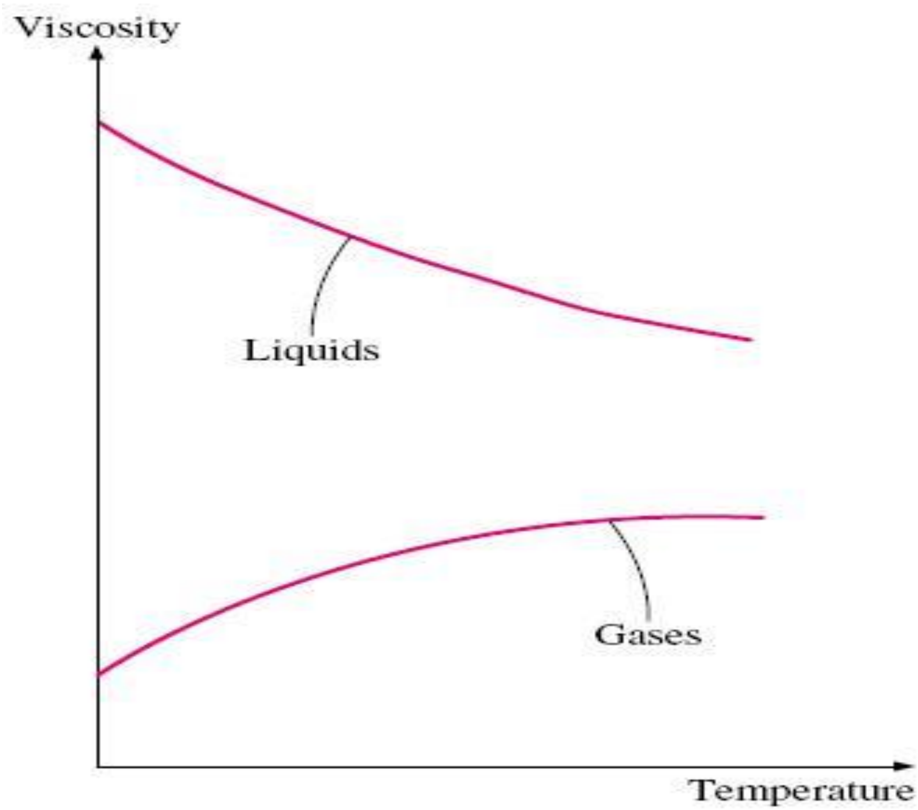
Variation of Viscosity with Temperature

Air at 20°C and 1 atm:

$$\mu = 1.83 \times 10^{-5}\ \text{kg/m} \cdot \text{s}$$
$$\upsilon = 1.52 \times 10^{-5}\ \text{m}^2/\text{s}$$

Air at 20°C and 4 atm:

$$\mu = 1.83 \times 10^{-5}\ \text{kg/m} \cdot \text{s}$$
$$\upsilon = 0.380 \times 10^{-5}\ \text{m}^2/\text{s}$$

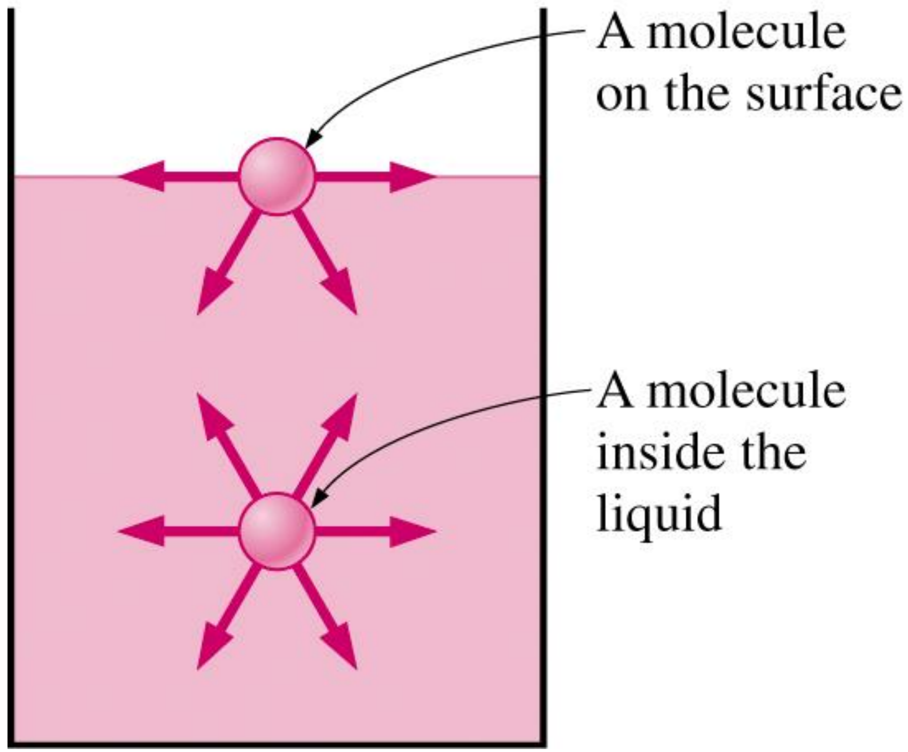


- In Liquids, viscosity is caused by the cohesive forces between the molecules.
- Viscosity of liquids decrease with increase in temperature. This is because in a liquid the molecules possess more energy at high temperature, so liquids can oppose cohesive intermolecular forces more strongly. As a result, the energized liquid molecules can move more freely

- In gases, Viscosity is caused by the molecular collisions between molecules.
- The intermolecular forces are negligible, so the gas molecules at high temperature move randomly at high velocities. As a result molecular collision per unit volume per unit time increases.

The viscosity of a fluid is directly related to the pumping power needed to transport a fluid in pipe or to move a body through a fluid.

Surface Tension



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Practical Examples

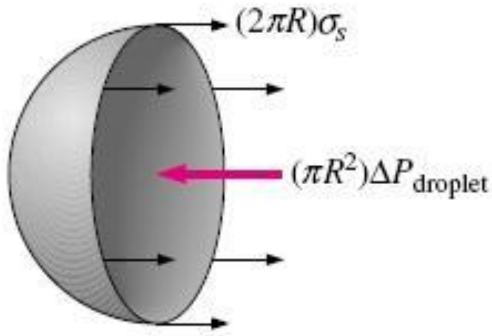
- Drop of blood forms a hump on a horizontal glass.
- Water droplets from rain
- A drop of mercury forms a near perfect square
- Dew hang from leaves of trees
- A soap released into air
- Liquid fuel injected into the engine

- In all these observations, the liquid droplets behave like small spherical balloons filled with liquid and the surface of the liquid acts like a stretched elastic membrane under tension.

- The pulling force that causes this tension acts parallel to the surface and it is due to cohesive forces between the molecules of the fluid.

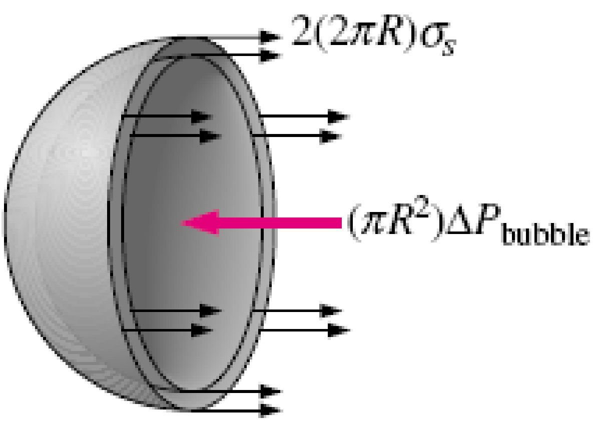
- Repulsive forces from interior molecules causes the liquid to minimize its surface area and attain a spherical shape

- The magnitude of this force per unit length is called surface tension σ_s (N/m). This effect is also called surface energy.



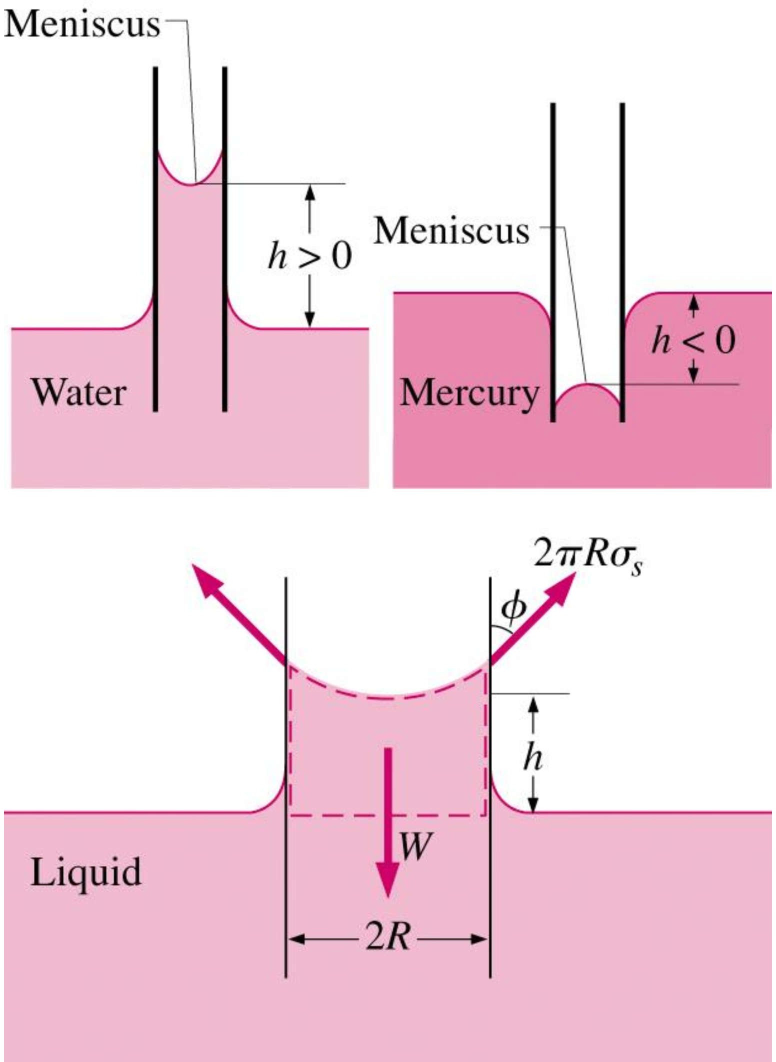
(*a*) Half a droplet

Droplet: $$(2\pi R)\sigma_s = (\pi R^2)\Delta P_{droplet} \rightarrow \Delta P_{droplet} = P_i - P_o = \frac{2\sigma_s}{R}$$



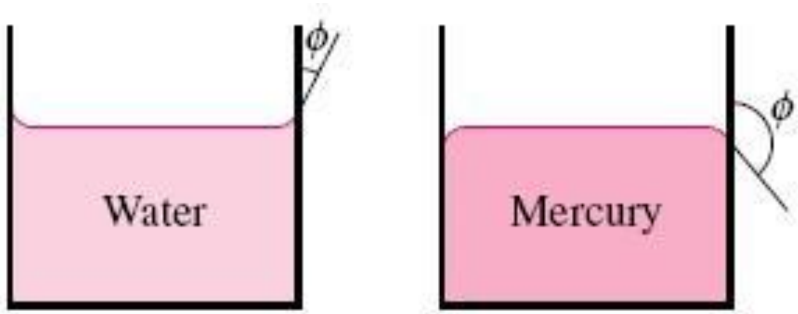
Bubble: $$2(2\pi R)\sigma_s = (\pi R^2)\Delta P_{bubble} \rightarrow \Delta P_{bubble} = P_i - P_o = \frac{4\sigma_s}{R}$$

Capillary Effect



- Capillary effect is the rise or fall of a liquid in a small-diameter tube.
- The curved free surface in the tube is called the meniscus.
- Water meniscus curves up because water is a *wetting fluid.*
- Mercury meniscus curves down because mercury is a *nonwetting fluid.*
- Force balance can describe magnitude of capillary rise.

Wetting or contact angle



(a) Wetting fluid

(b) Nonwetting fluid

The strength of capillary effect is quantified by contact angle

It is defined as the angle that the tangent to the liquid surface makes with solid surface at the point of contact

A liquid is said to wet the surface if $\phi < 90°$ and

not to wet the surface when $\phi > 90°$

Capillary rise/drop

$$h = 2\sigma_s \cos\phi / \rho g R$$

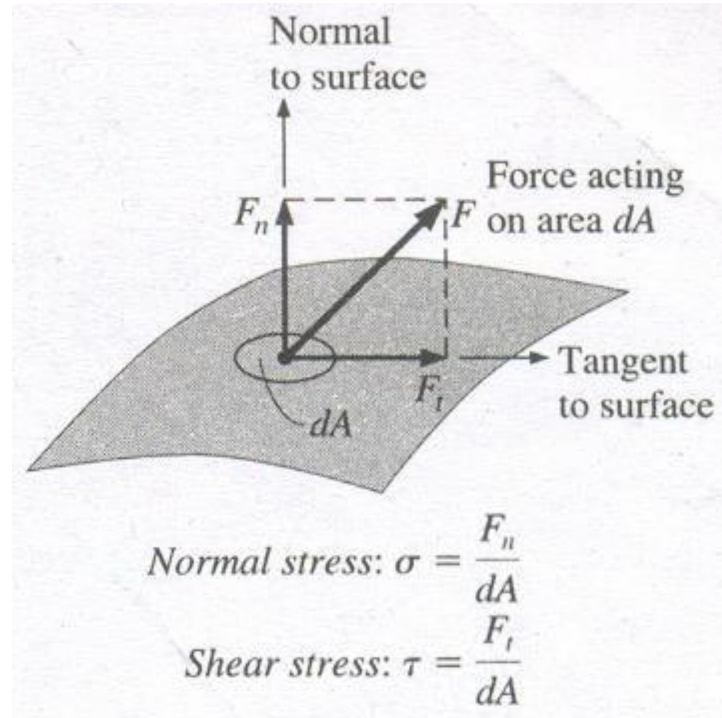
Pressure

- Pressure is defined as a normal force exerted by a fluid per unit area.

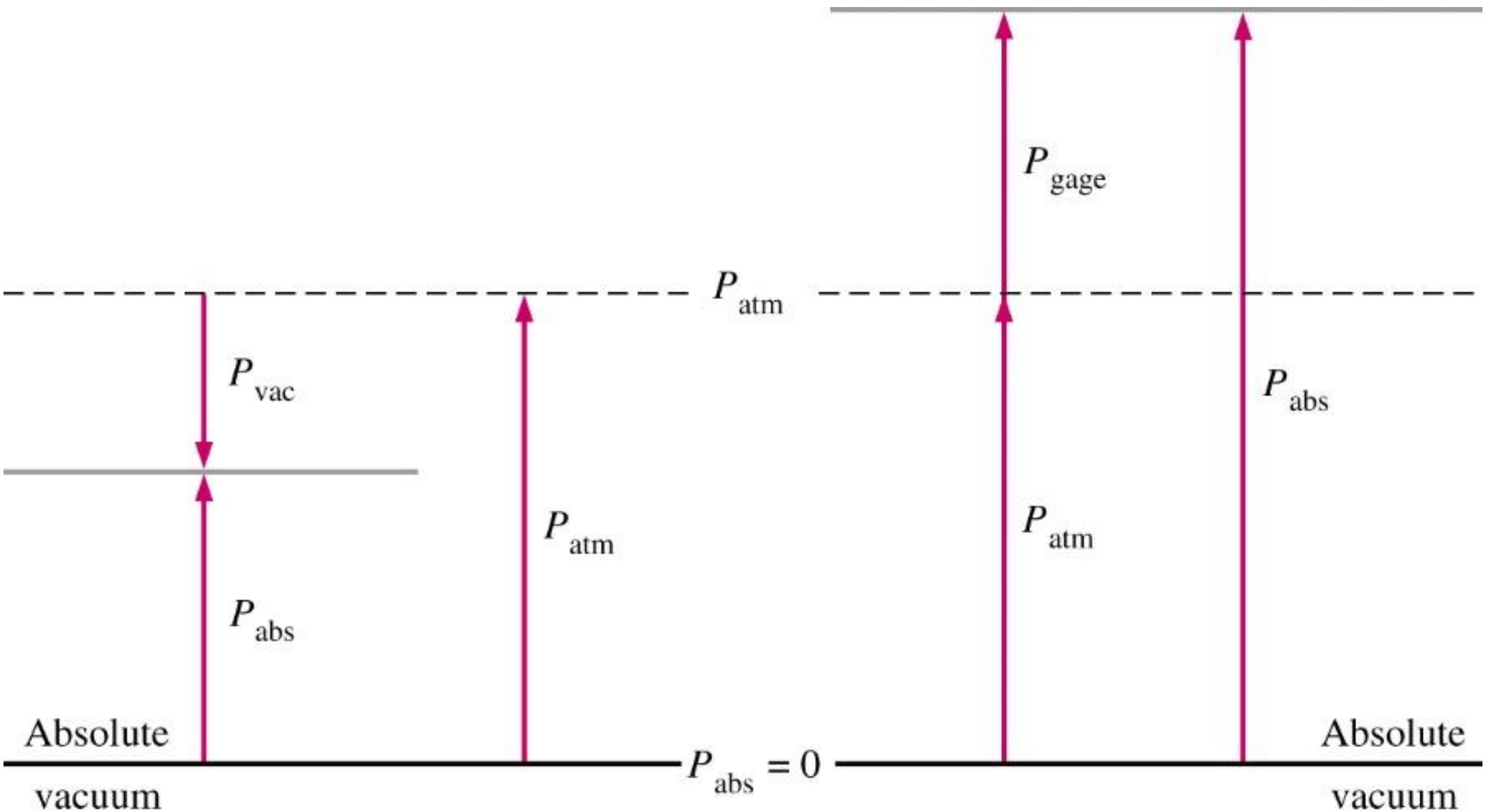
- Unit of pressure is N/m^2, which is also called as pascal (Pa).

- Since the unit Pa is too small for pressures encountered in practice, kilopascal (1 kPa = 10^3 Pa) and megapascal (1 MPa = 10^6 Pa) are commonly used.

- Other units include bar, atm, kgf/cm^2, lbf/in^2=psi.



Absolute, gage, and vacuum pressures

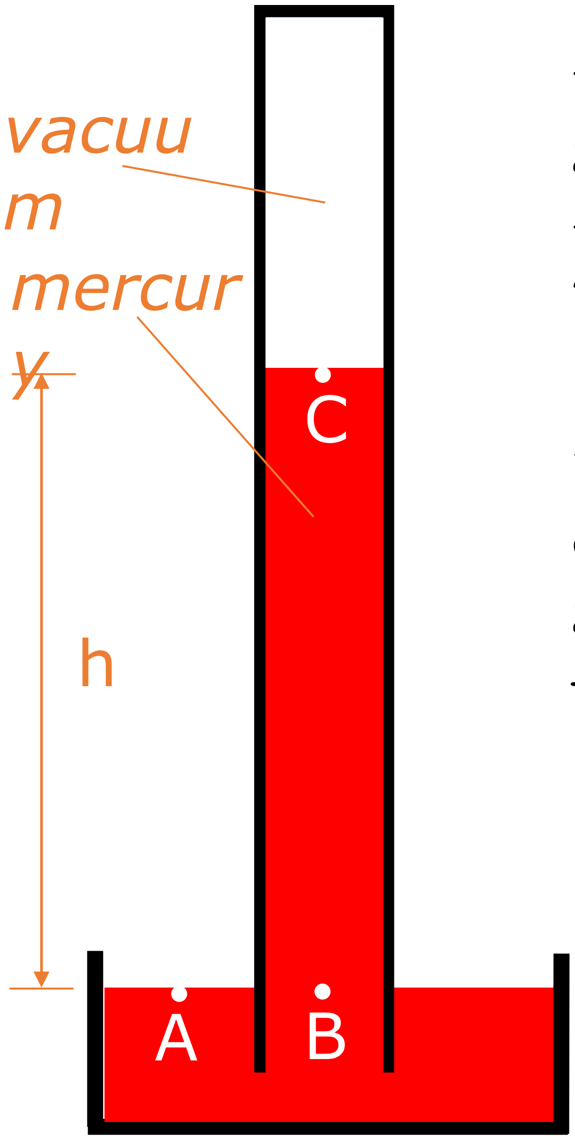


- Actual pressure at a give point is called the absolute pressure.

- Most pressure-measuring devices are calibrated to read zero in the atmosphere, and therefore indicate gage pressure, $P_{gage}=P_{abs} - P_{atm}$.

- Pressure below atmospheric pressure are called vacuum pressure, $P_{vac}=P_{atm} - P_{abs}$.

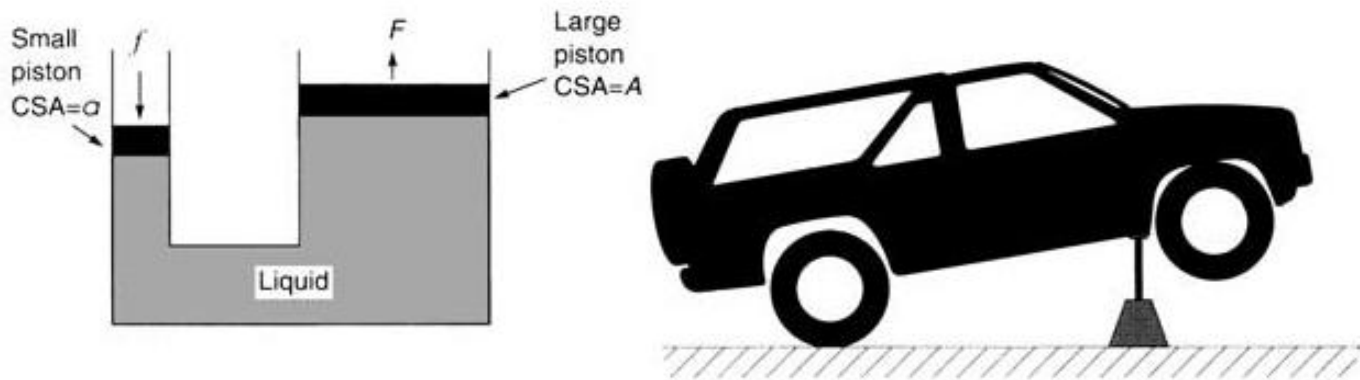
Barometers



The pressure at A is the same as the pressure of the surrounding air, since it's at the surface. A and B are at the same pressure, since they are at the same height. The pressure at C is zero, since a vacuum has no pressure. The pressure difference from B to C is $\rho g h$ (where ρ is the density of mercury), which is the pressure at B, which is the pressure at A, which is the air pressure. Thus, the height of the barometer directly measures air pressure.

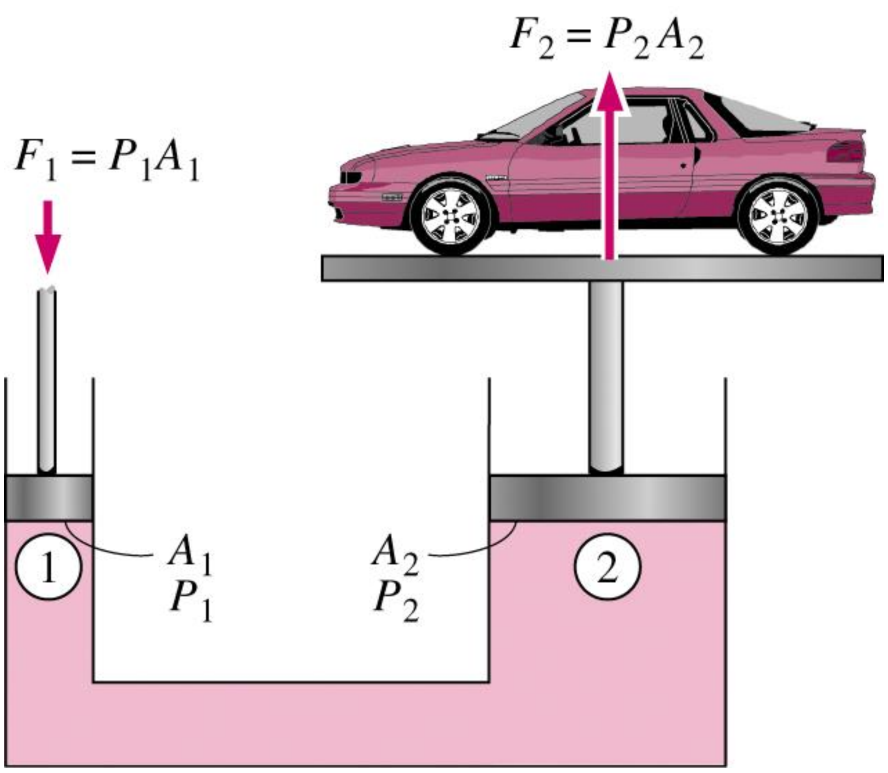
Pressure at a Point

- Pressure at any point in a fluid is the same in all directions. (Pascal's Law) $P_x = P_y = P_z$
- Pressure has a magnitude, but not a specific direction, and thus it is a scalar quantity.



Hydraulic Jack

Pascal's Law

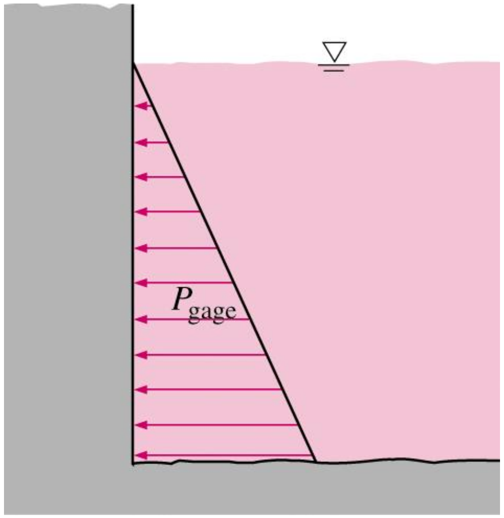


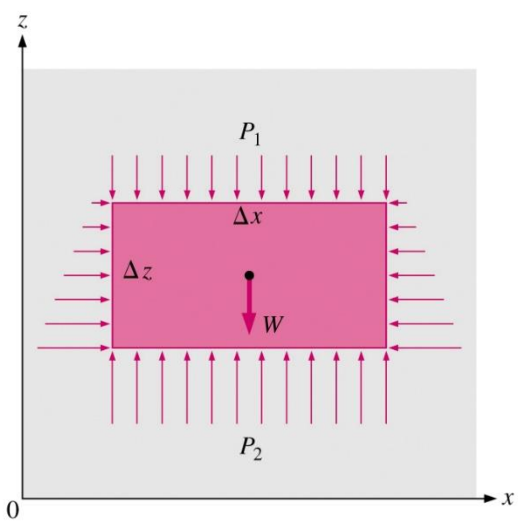
- Pressure applied to a confined fluid increases the pressure throughout by the same amount.
- In picture, pistons are at same height:

$$P_1 = P_2 \rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2} \rightarrow \frac{F_2}{F_1} = \frac{A_2}{A_1}$$

- Ratio A2/A1 is called ideal mechanical advantage

Variation of Pressure with Depth





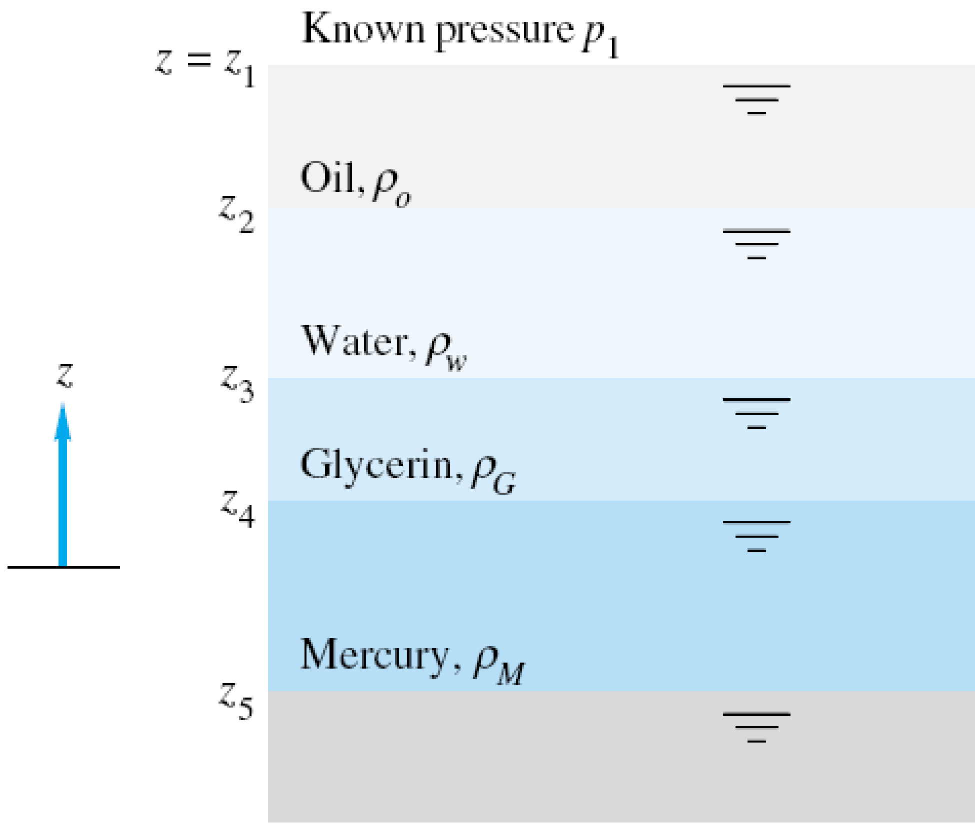
- In the presence of a gravitational field, pressure increases with depth because more fluid rests on deeper layers.
- To obtain a relation for the variation of pressure with depth, consider rectangular element
  - Force balance in *z*-direction gives

$$\sum F_z = ma_z = 0$$

$$P_2\Delta x - P_1\Delta x - \rho g \Delta x \Delta z = 0$$

$$\Delta P = P_2 - P_1 = \rho g \Delta z = \gamma_s \Delta z$$

Evaluating Pressure changes through a column of multiple fluids



$$p_2 - p_1 = -\rho_o g(z_2 - z_1)$$

$$p_3 - p_2 = -\rho_w g(z_3 - z_2)$$

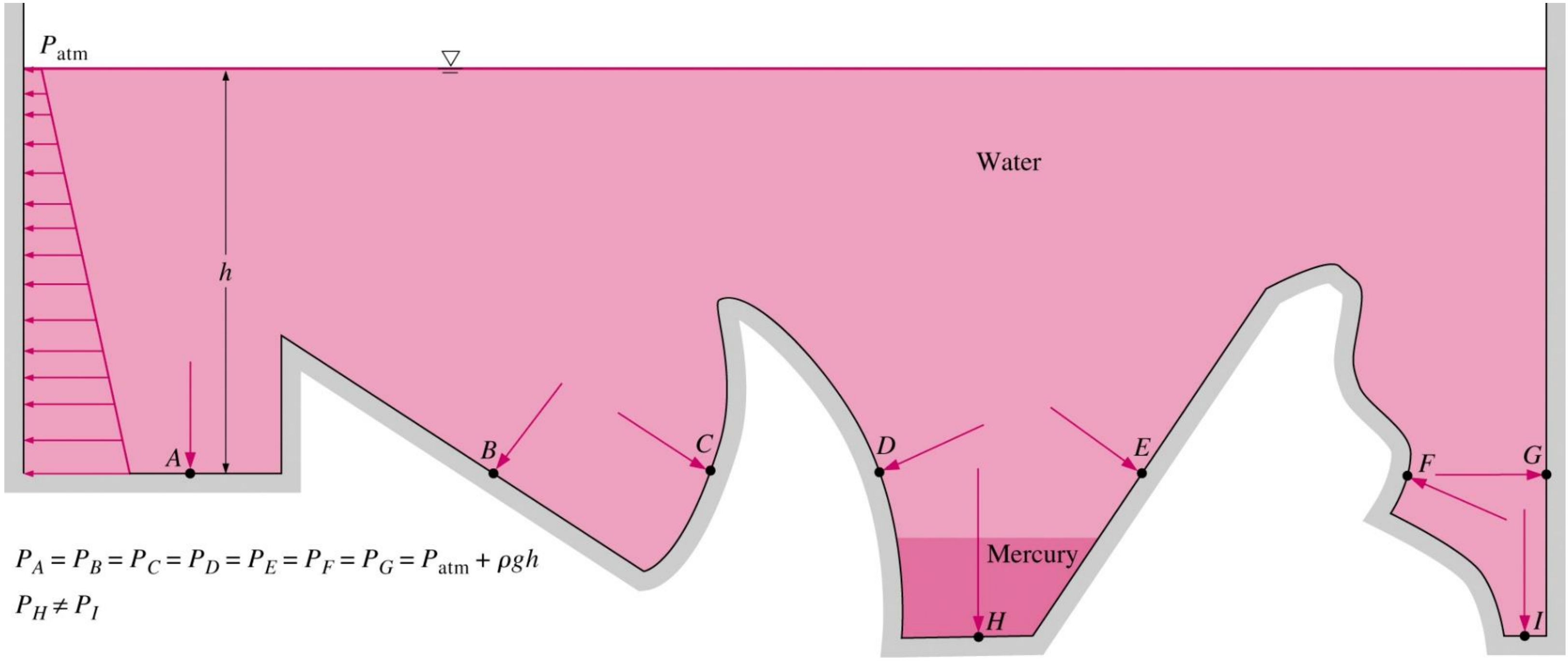
$$p_4 - p_3 = -\rho_G g(z_4 - z_3)$$

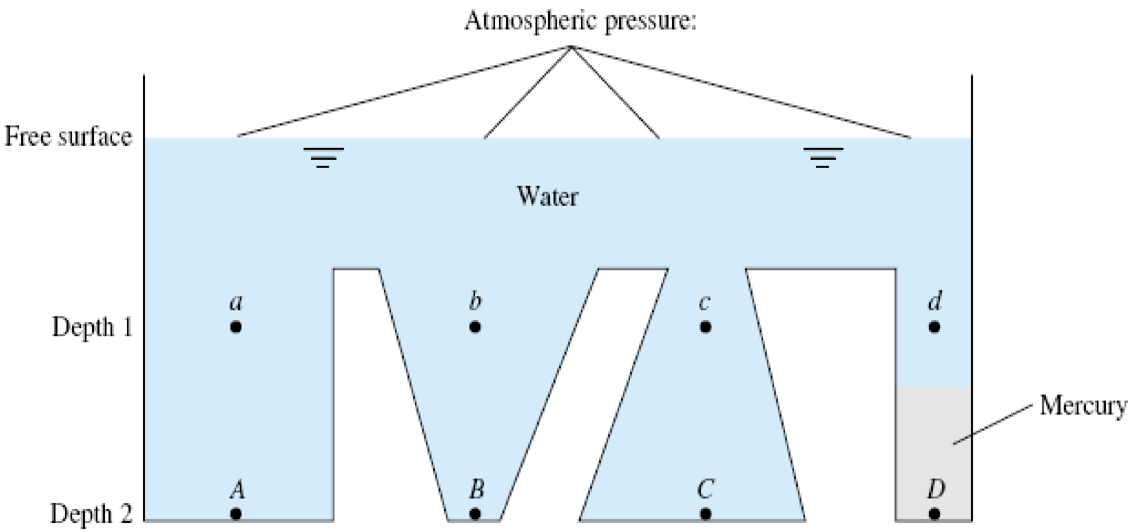
$$p_5 - p_4 = -\rho_M g(z_5 - z_4)$$

$$\text{Sum} = p_5 - p_1$$

Variation of Pressure with Depth

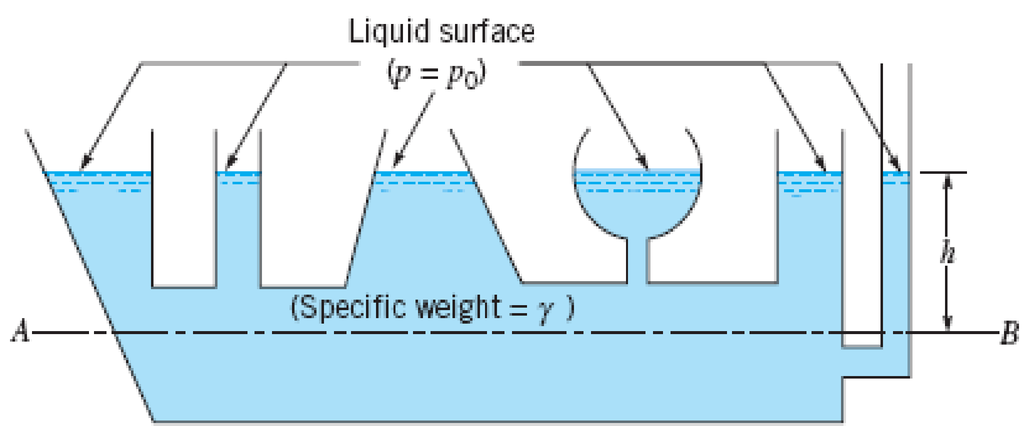
- Pressure in a fluid at rest is independent of the shape of the container.
- Pressure is the same at all points on a horizontal plane in a given fluid.





Points a, b, c, and d are at equal depths in water and therefore have identical pressures.

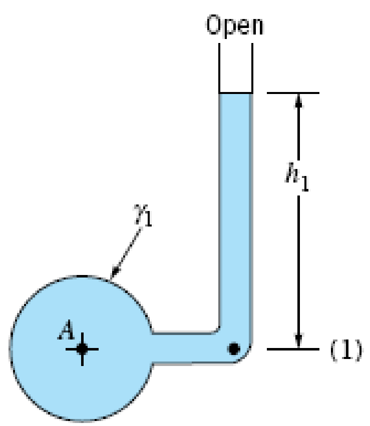
Point D has a different pressure from A, B, and C because it is not connected to them by a water path



Manometry

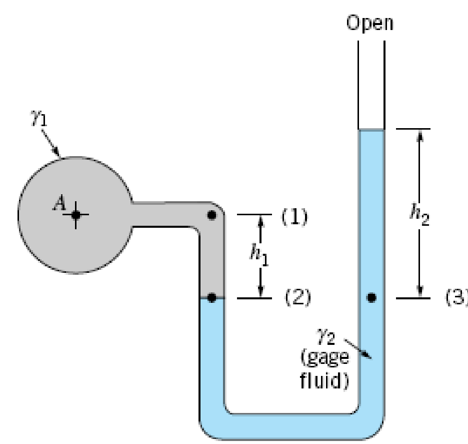
Pressure measuring devices based on liquid columns in vertical or inclined tubes are called manometers

Piezometer Tube



$$p_A = \gamma_1 h_1$$

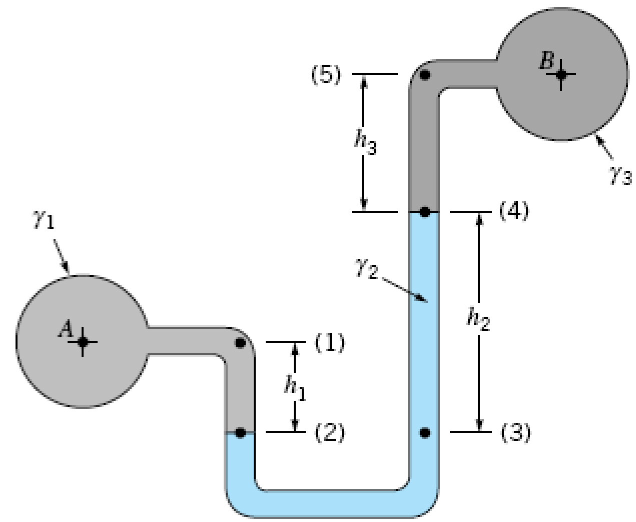
U-Tube Manometer

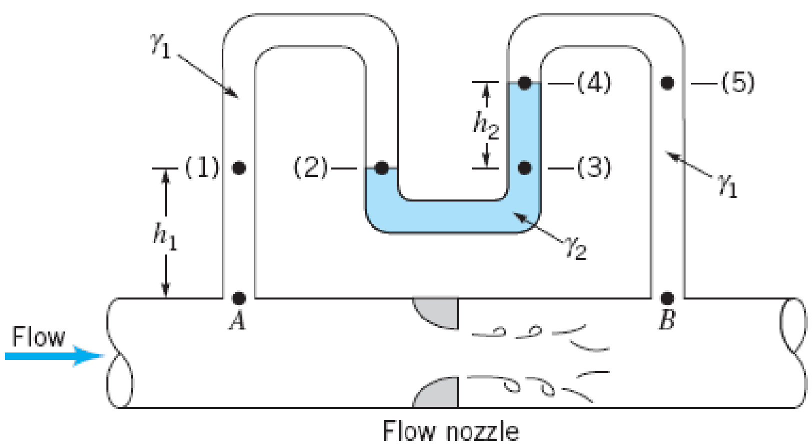


$$p_A + \gamma_1 h_1 - \gamma_2 h_2 = 0$$

Differential U-Tube Manometer

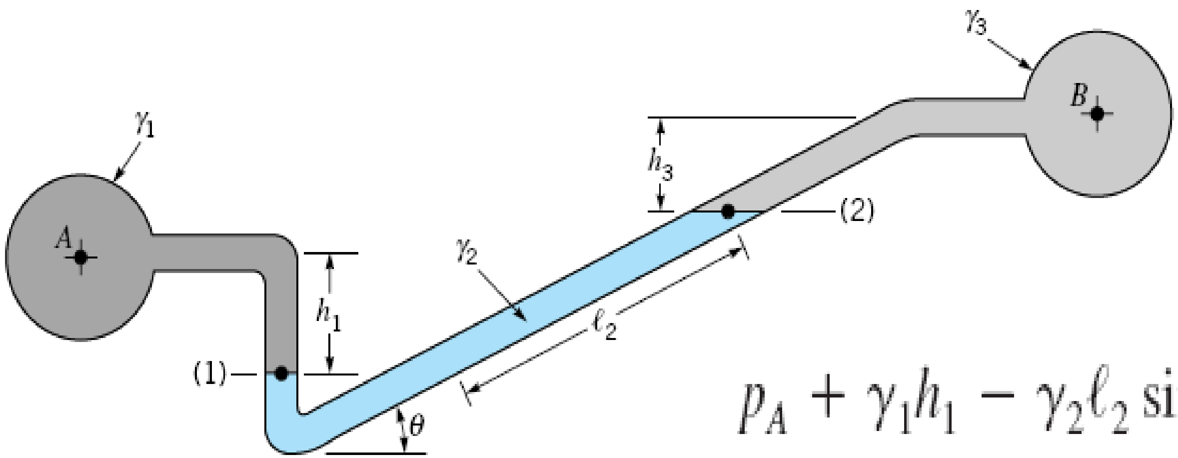
$$p_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = p_B$$





$$p_A - \gamma_1 h_1 - \gamma_2 h_2 + \gamma_1(h_1 + h_2) = p_B$$

Inclined-Tube Manometer



$$p_A + \gamma_1 h_1 - \gamma_2 \ell_2 \sin\theta - \gamma_3 h_3 = p_B$$

Inclined-tube manometers can be used to measure small pressure differences accurately.

Type of Fluids

Classification of Flows

Viscous vs. Inviscid Regions of Flow

Internal vs. External Flow

Compressible vs. Incompressible Flow

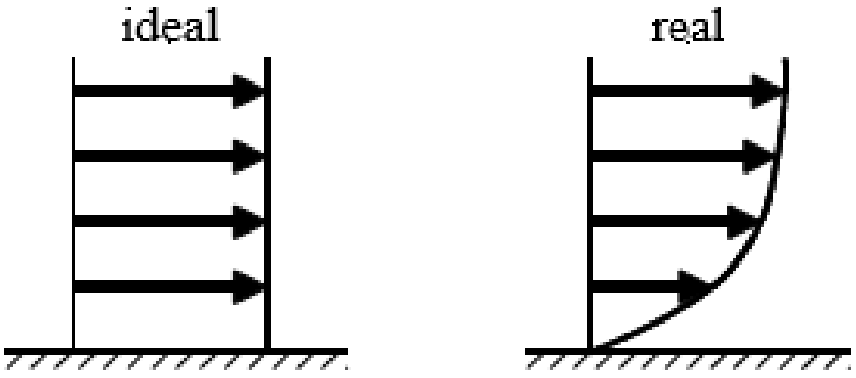
Laminar vs. Turbulent Flow

Steady vs. Unsteady Flow

One-, Two-, and Three-Dimensional Flows

Ideal fluids & Real Fluids

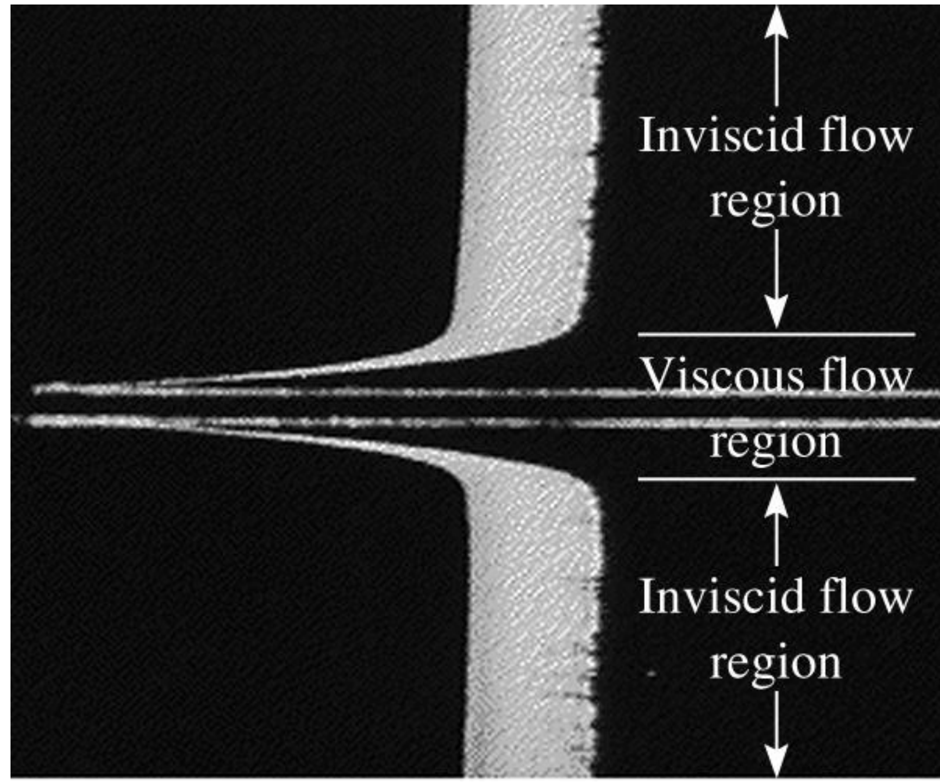
- Ideal fluids have no viscosity – there is no internal friction or loss of mechanical energy.
- No such fluid exists, but many flows can be approximated as ideal if viscous forces are small and do not cause major flow phenomena such as boundary-layer separation.
- Real fluids have non-zero viscosity
- They satisfy the no-slip condition at solid boundaries. i.e. the (relative) velocity at the boundary is zero.



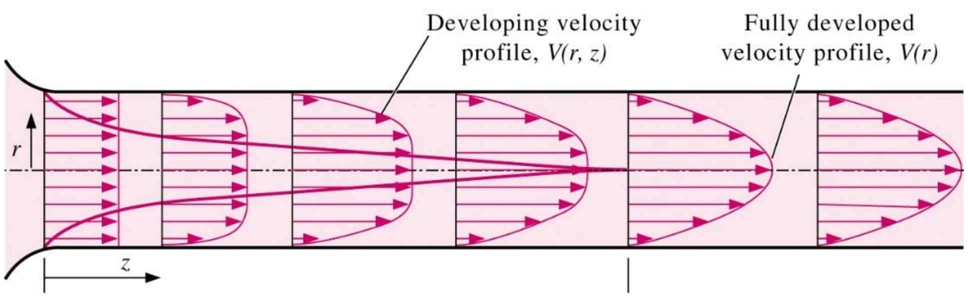
Viscous vs. Inviscid Regions of Flow

- Regions where frictional effects are significant are called viscous regions. They are usually close to solid surfaces.

- Regions where frictional forces are small compared to inertial or pressure forces are called inviscid

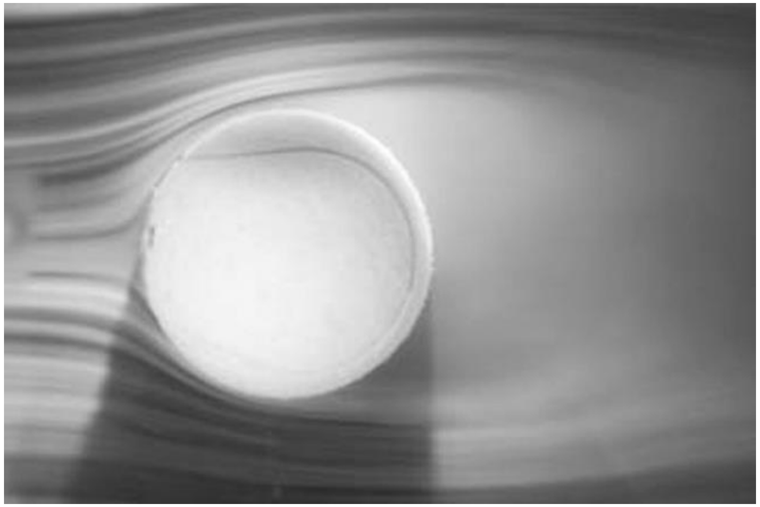


Internal vs. External Flow



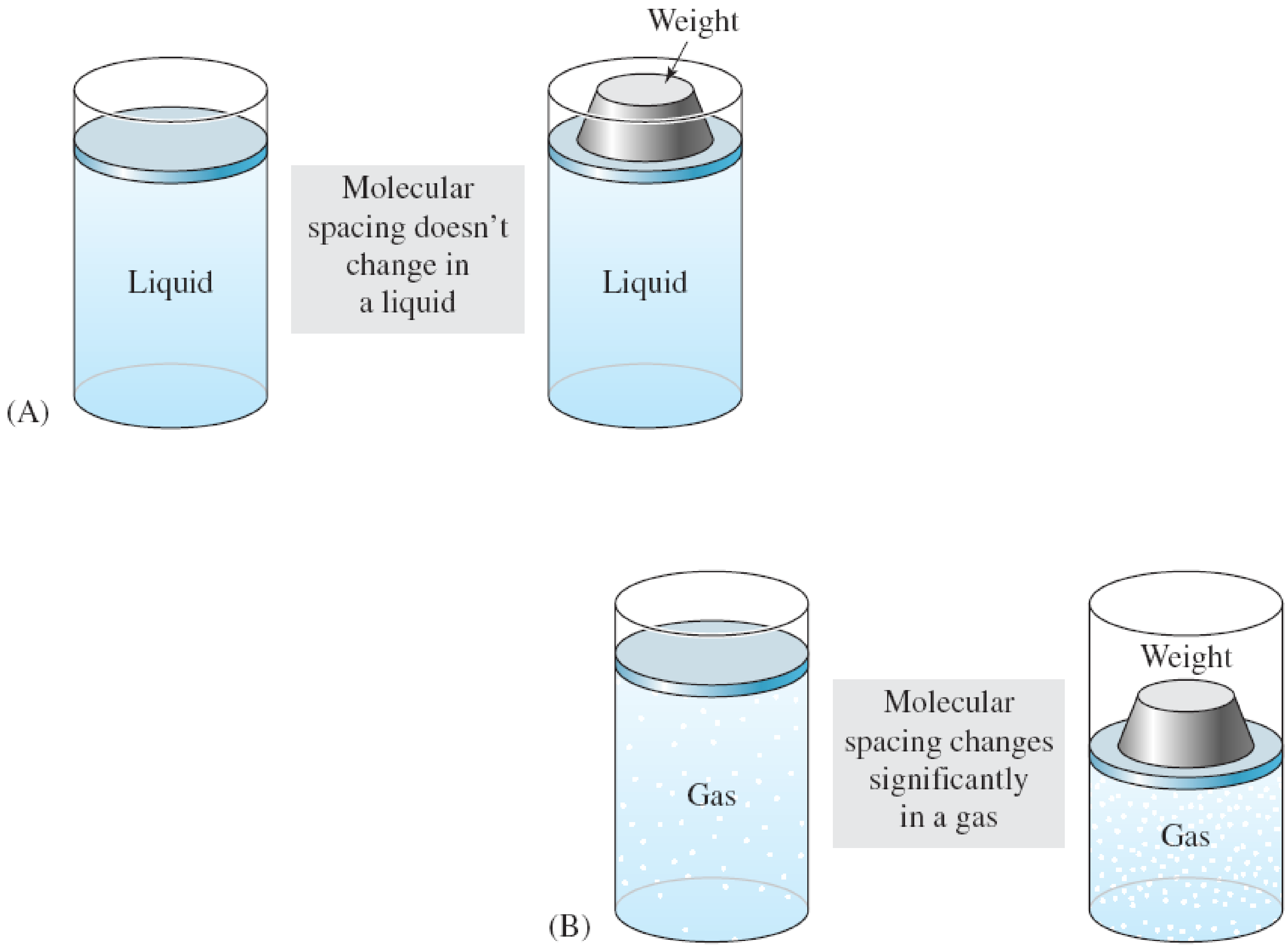
- Internal flows are dominated by the influence of viscosity throughout the flow field

- For external flows, viscous effects are limited to the boundary layer and wake.



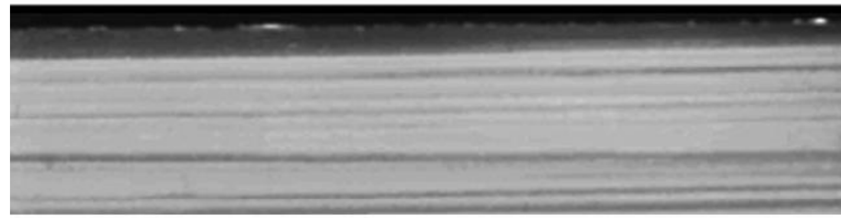
Compressible vs. Incompressible Flow

- A flow is classified as incompressible if the density remains nearly constant.
- Liquid flows are typically incompressible.
- Gas flows are often compressible, especially for high speeds.
- Mach number, $Ma = c / a$ is a good indicator of whether or not compressibility effects are important.
  - $Ma < 0.3$: Incompressible
  - $Ma < 1$: Subsonic
  - $Ma = 1$: Sonic
  - $Ma > 1$: Supersonic
  - $Ma \gg 1$: Hypersonic

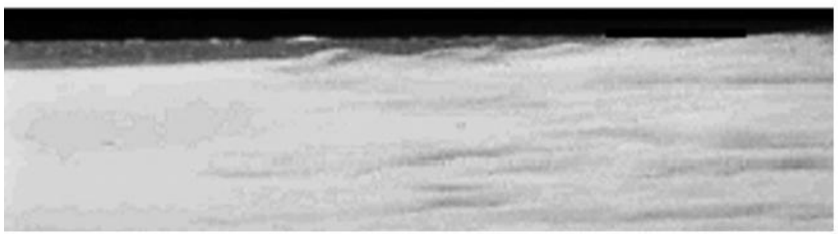

Weight
Liquid
Molecular spacing doesn't change in a liquid
Liquid
(A)
Gas
Molecular spacing changes significantly in a gas
Weight
Gas
(B)

Laminar vs. Turbulent Flow

Laminar: highly ordered fluid motion with smooth streamlines.



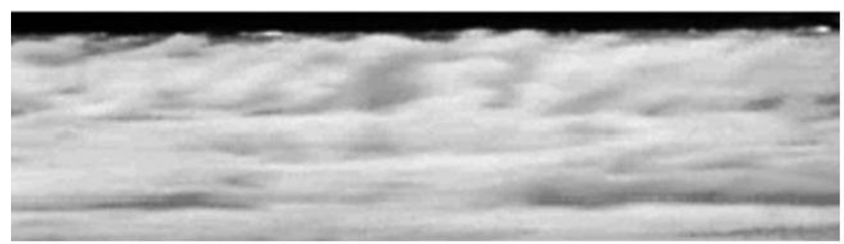
Laminar



Transitional

Transitional: a flow that contains both laminar and turbulent regions

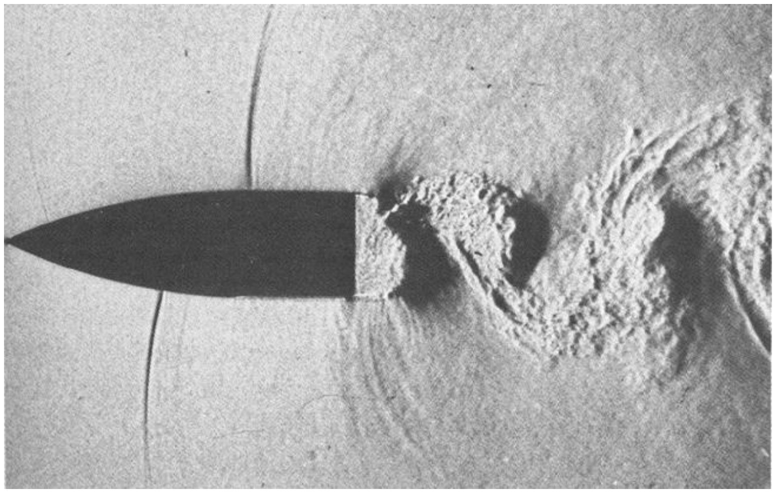
Turbulent: highly disordered fluid motion characterized by velocity fluctuations and eddies.

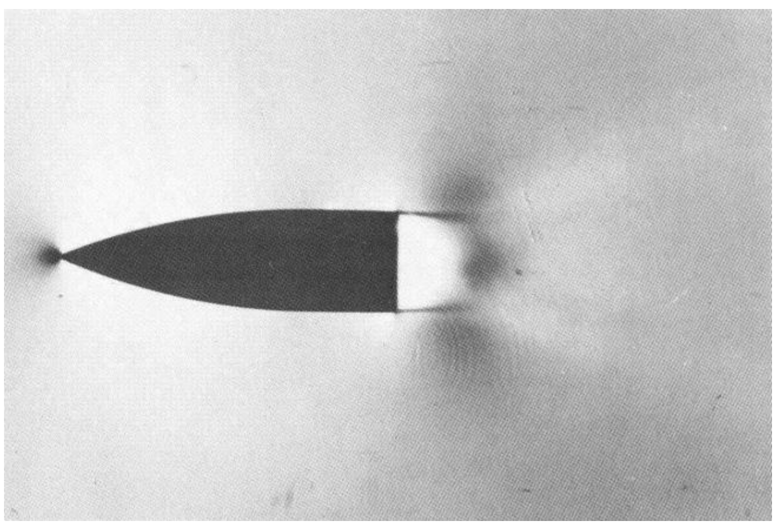


Turbulent

Reynolds number, is the key parameter in determining whether flow is laminar or turbulent.

Steady vs. Unsteady Flow





- Steady implies no change at a point with time.

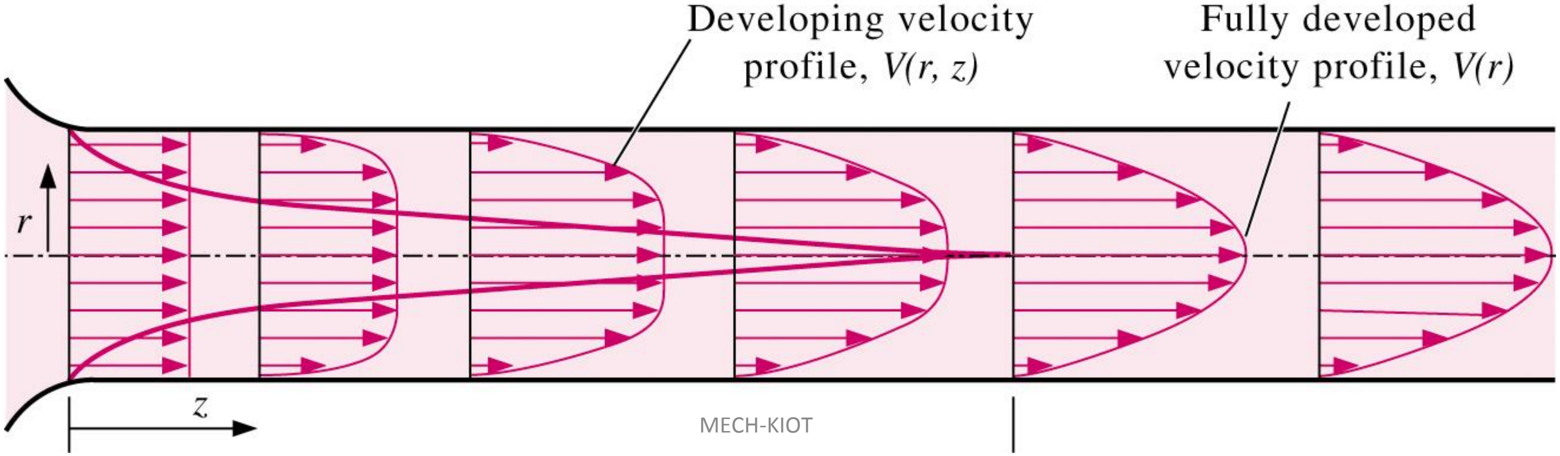
$$\frac{\partial \mathbf{U}}{\partial t} = \frac{\partial \rho}{\partial t} = 0$$

- Unsteady is the opposite of steady.
  - Transient usually describes a starting, or developing flow.
  - Periodic refers to a flow which oscillates about a mean.

- Unsteady flows may appear steady if "time-averaged"

One-, Two-, and Three-Dimensional Flows

- Velocity vector, U(x,y,z,t)= [Ux(x,y,z,t),Uy(x,y,z,t),Uz(x,y,z,t)]
- Lower dimensional flows reduce complexity of analytical and computational solution
- Change in coordinate system (cylindrical, spherical, etc.) may facilitate reduction in order.
- Example: for fully-developed pipe flow, velocity V(r) is a function of radius r and pressure p(z) is a function of distance z along the pipe.

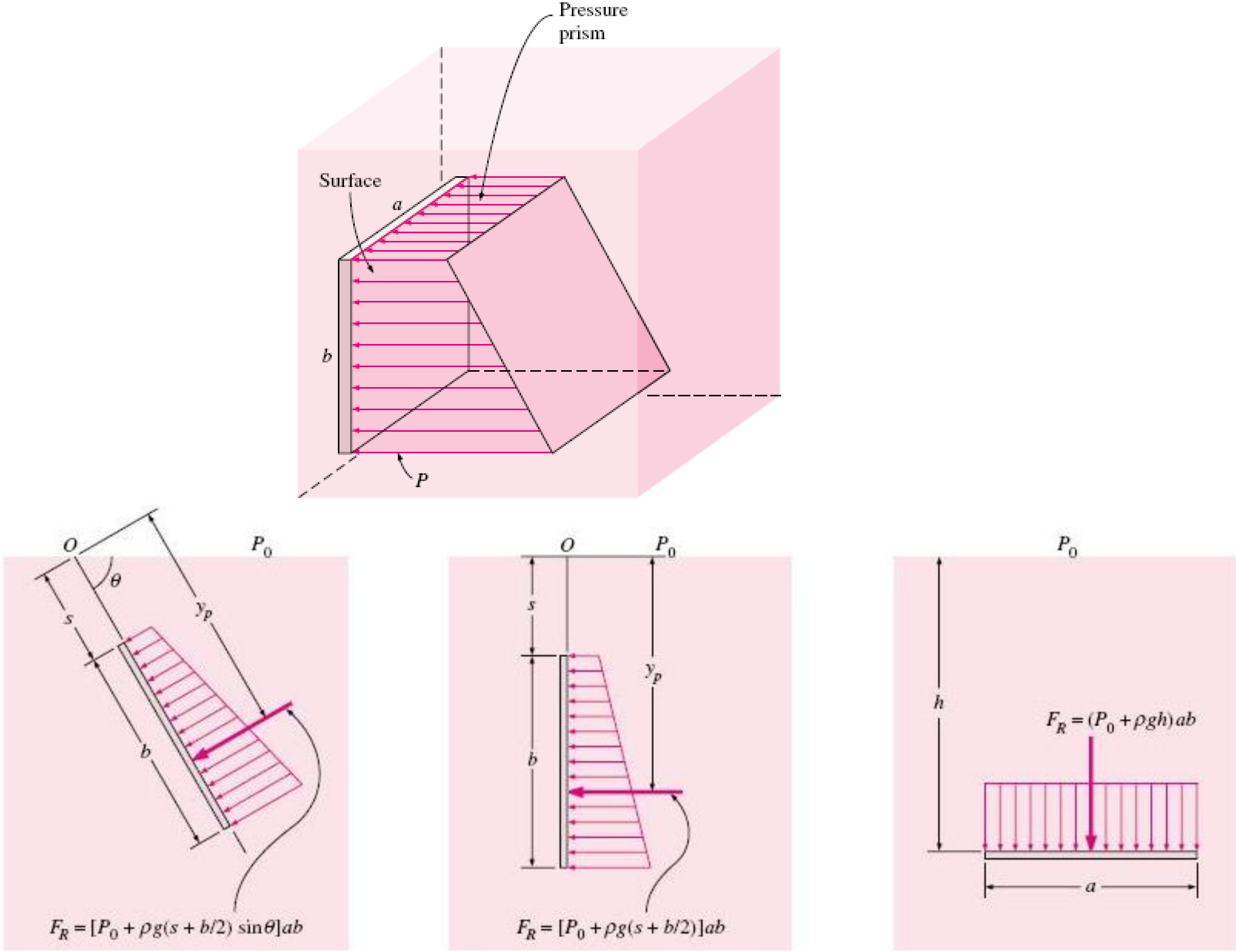


Fluid Statics

Chapter 03

Fluid Statics

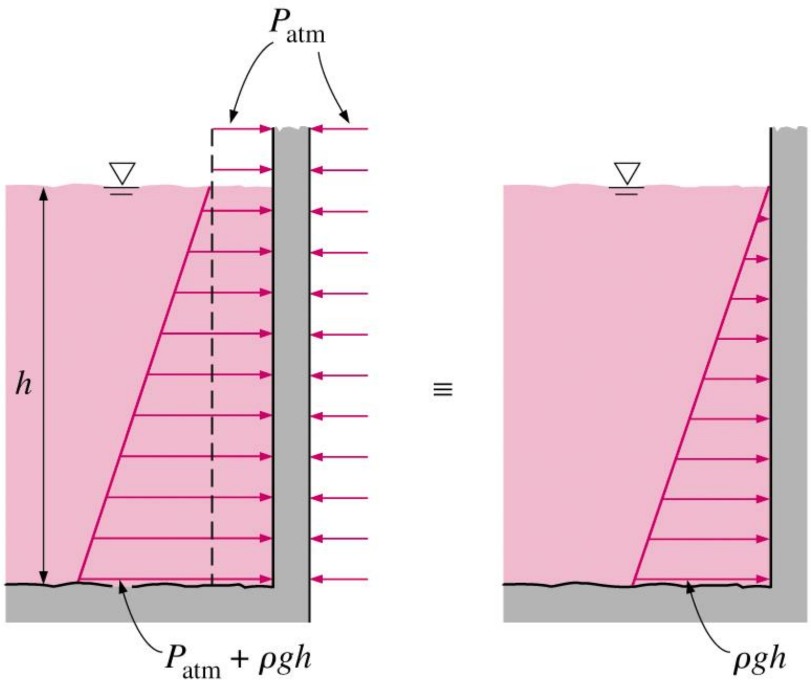
- Fluid Statics deals with problems associated with fluids at rest.
- In fluid statics, there is no relative motion between adjacent fluid layers.
- Therefore, there is no shear stress in the fluid trying to deform it.
- The only stress in fluid statics is *normal stress*
  - Normal stress is due to pressure
- Applications: Floating or submerged bodies, water dams and gates, liquid storage tanks, etc.



Pressure prism
Surface
a
b
P
O
P_0
θ
s
y_p
b
$F_R = [P_0 + \rho g(s + b/2) \sin\theta]ab$
O
P_0
s
y_p
b
$F_R = [P_0 + \rho g(s + b/2)]ab$
P_0
h
$F_R = (P_0 + \rho gh)ab$
a

Hoover Dam

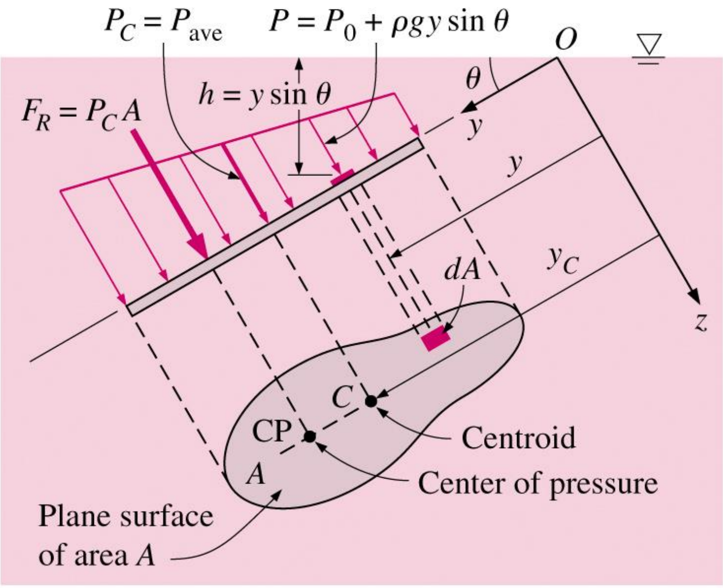
Hydrostatic Forces on Plane Surfaces

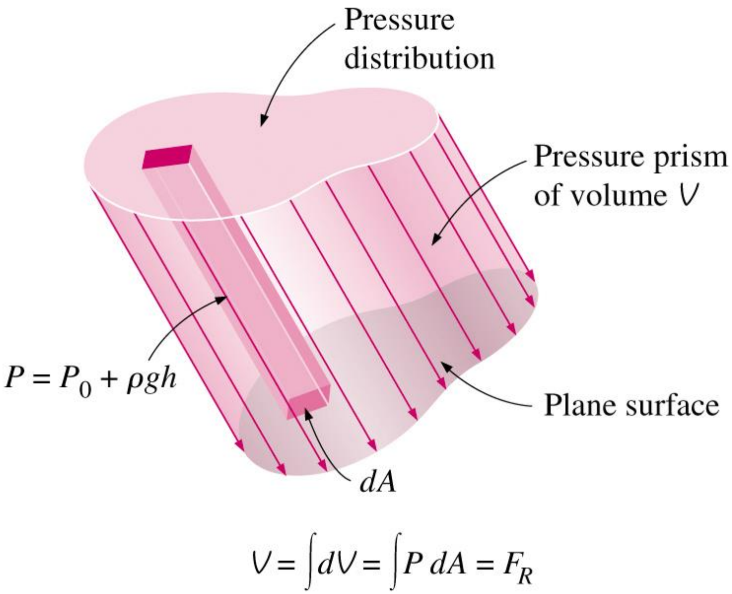


(a) P_{atm} considered   (b) P_{atm} subtracted

- On a plane surface, the hydrostatic forces form a system of parallel forces
- For many applications, magnitude and location of application, which is called center of pressure, must be determined.
- Atmospheric pressure P_{atm} can be neglected when it acts on both sides of the surface.

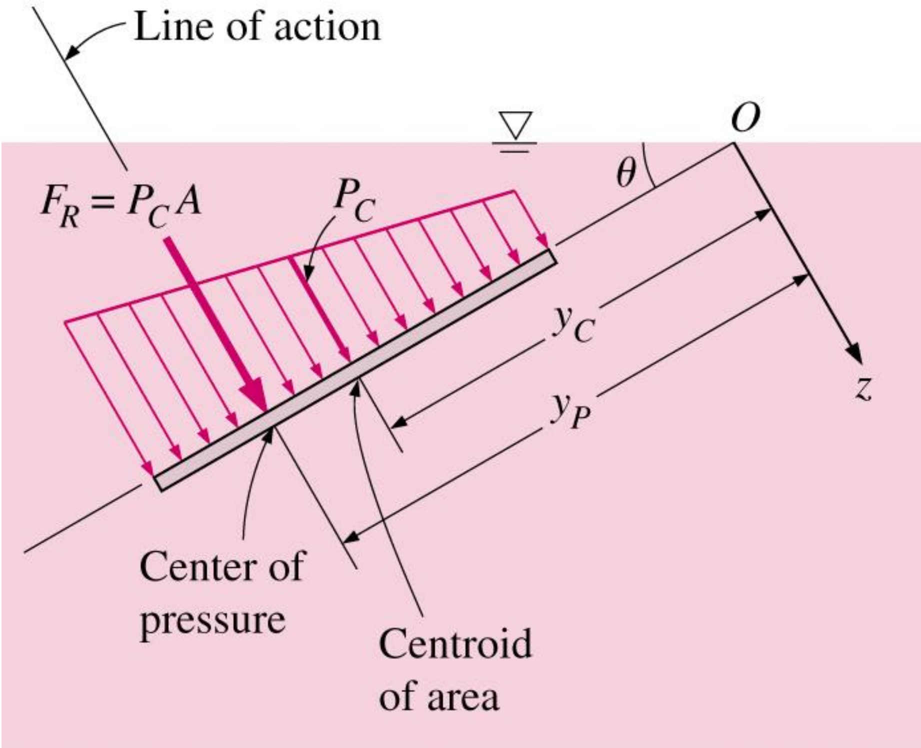
Resultant Force





- The magnitude of F_R acting on a plane surface of a completely submerged plate in a homogenous fluid is equal to the product of the pressure P_C at the centroid of the surface and the area A of the surface

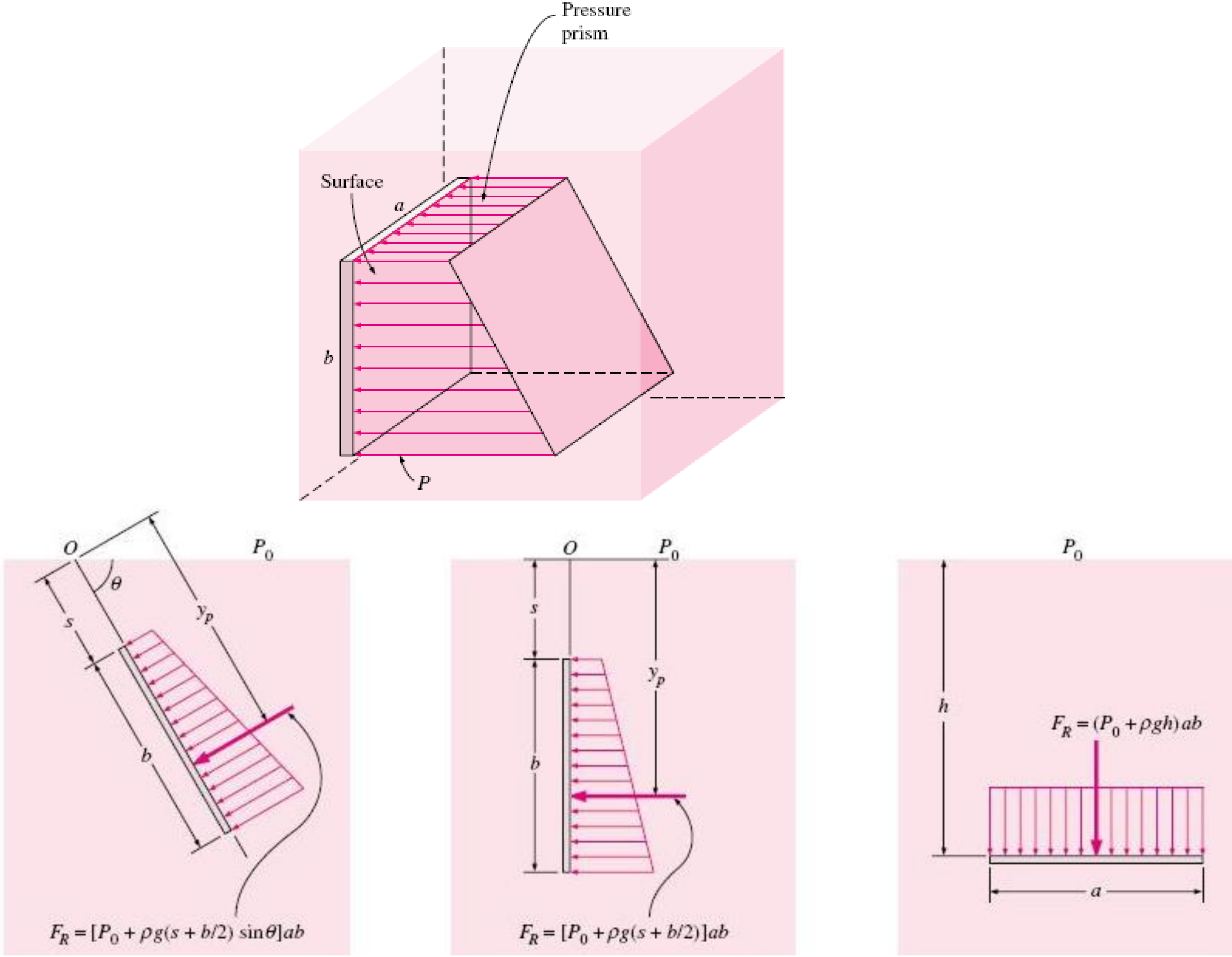
Center of Pressure

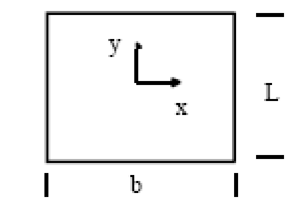
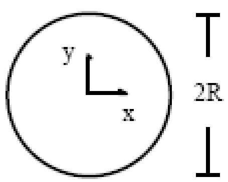
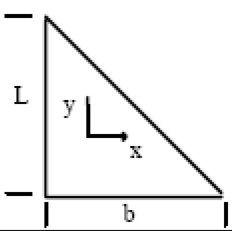
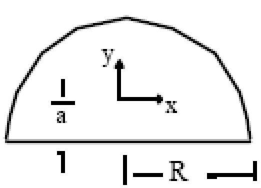


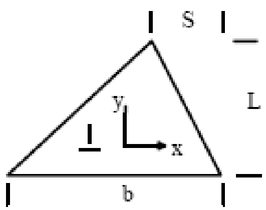
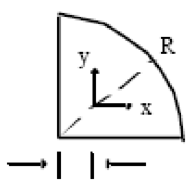
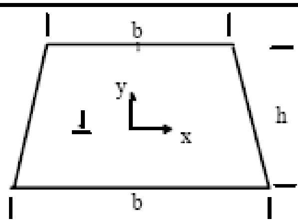
- Line of action of resultant force $F_R=P_CA$ does not pass through the centroid of the surface. In general, it lies underneath where the pressure is higher.
- Vertical location of Center of Pressure is determined by equating the moment of the resultant force to the moment of the distributed pressure force.

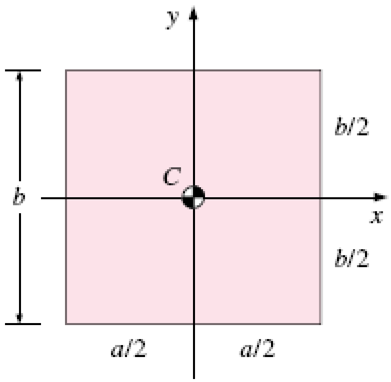
$$y_p = y_C + \frac{I_{xx,C}}{y_c A}$$

- $I_{xx,C}$ is tabulated for simple geometries.


Pressure prism
Surface
a
b
P
O
P0
θ
s
yp
b
FR = [P0 + ρg(s + b/2) sinθ]ab
O
P0
s
yp
b
FR = [P0 + ρg(s + b/2)]ab
P0
h
FR = (P0 + ρgh)ab
a

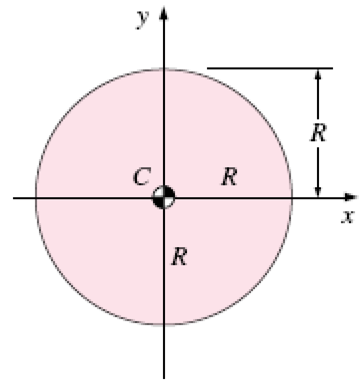
Geometry	Centroid	Moment of Inertia I_{xx}	Product of Inertia I_{xy}	Area
	$b/2, L/2$	$\frac{bL^3}{12}$	0	$b \cdot L$
	0, 0	$\frac{\pi R^4}{4}$	0	πR^2
	$b/3, \quad L/3$	$\frac{bL^3}{36}$	$-\frac{b^2L^2}{72}$	$\frac{b \cdot L}{2}$
	$0, a = \frac{4R}{3\pi}$	$R^4\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)$	0	$\frac{\pi R^2}{2}$

Geometry	Centroid	Moment of Inertia I_{xx}	Product of Inertia I_{xy}	Area
	$a = \frac{L}{3}$	$\frac{bL^3}{36}$	$\frac{b(b-2s)L^2}{72}$	$\frac{1}{2} b \cdot L$
	$a = \frac{4R}{3\pi}$	$\left(\frac{\pi}{16} - \frac{4}{9\pi}\right) R^4$	$\left(\frac{1}{8} - \frac{4}{9\pi}\right) R^4$	$\frac{\pi R^2}{4}$
	$a = \frac{h(b+2b_1)}{3(b+b_1)}$	$\frac{h^3(b^2 + 4bb_1 + b_1^2)}{36(b+b_1)}$	0	$(b+b_1)\frac{h}{2}$



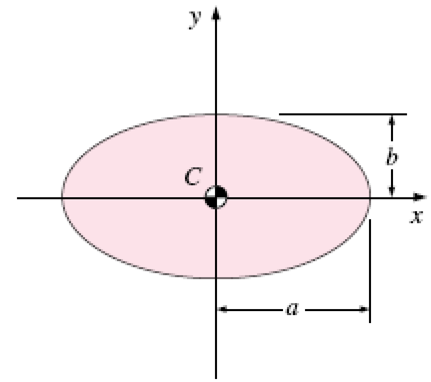
$A = ab,\ I_{xx,C} = ab^3/12$

(*a*) Rectangle



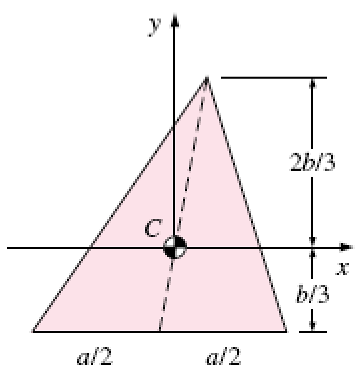
$A = \pi R^2,\ I_{xx,C} = \pi R^4/4$

(*b*) Circle



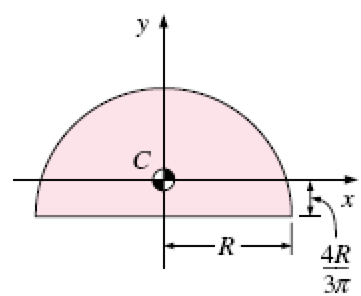
$A = \pi ab,\ I_{xx,C} = \pi ab^3/4$

(*c*) Ellipse



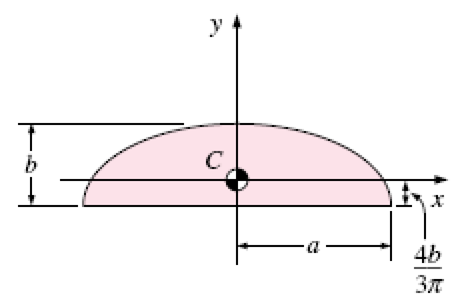
$A = ab/2,\ I_{xx,C} = ab^3/36$

(*d*) Triangle



$A = \pi R^2/2,\ I_{xx,C} = 0.109757R^4$

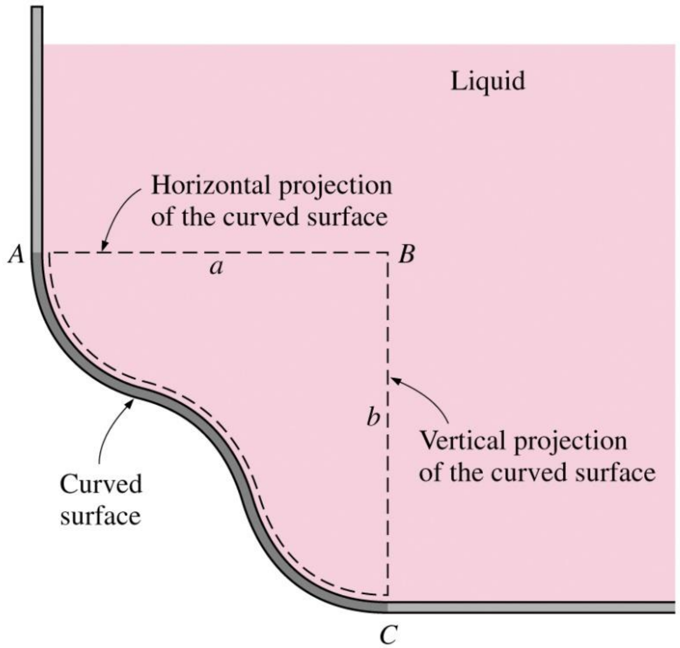
(*e*) Semicircle

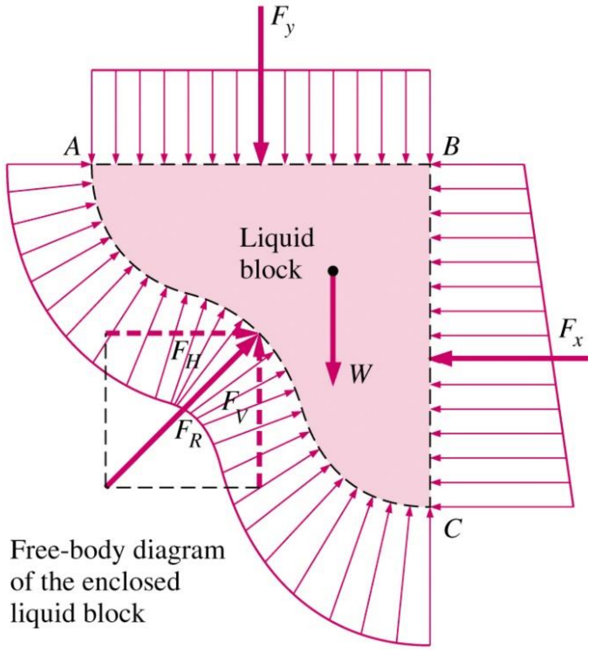


$A = \pi ab/2,\ I_{xx,C} = 0.109757ab^3$

(*f*) Semiellipse

Hydrostatic Forces on Curved Surfaces





- F_R on a curved surface is more involved since it requires integration of the pressure forces that change direction along the surface.
- Easiest approach: determine horizontal and vertical components F_H and F_V separately.

Horizontal force component on the curved surface : $F_H = F_x$

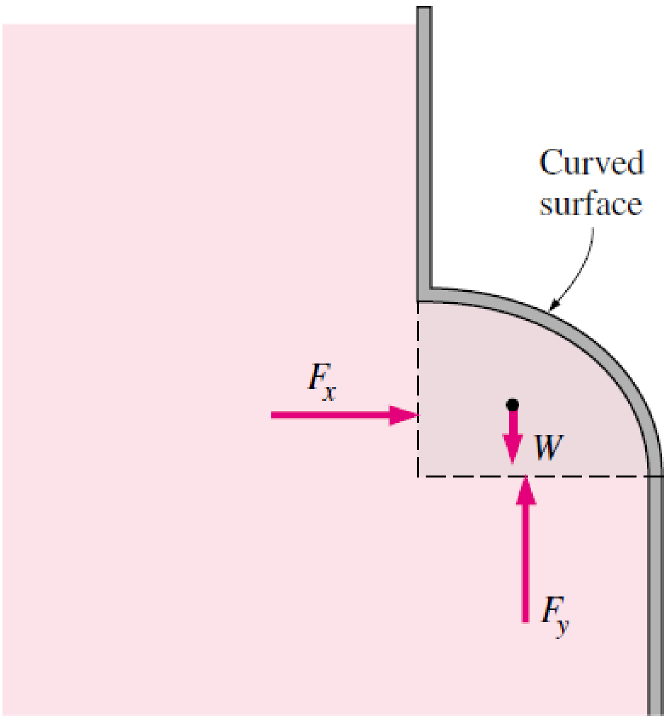
Vertical force component on the curved surface:

$F_V = F_y \pm w$

Magnitude of Resultant hydrostatic force

$$\mathbf{F_R} = \sqrt{\mathbf{F_H}^2 + \mathbf{F_V}^2}$$

$$\tan\alpha = \left(\frac{\mathbf{F_V}}{\mathbf{F_H}}\right)$$



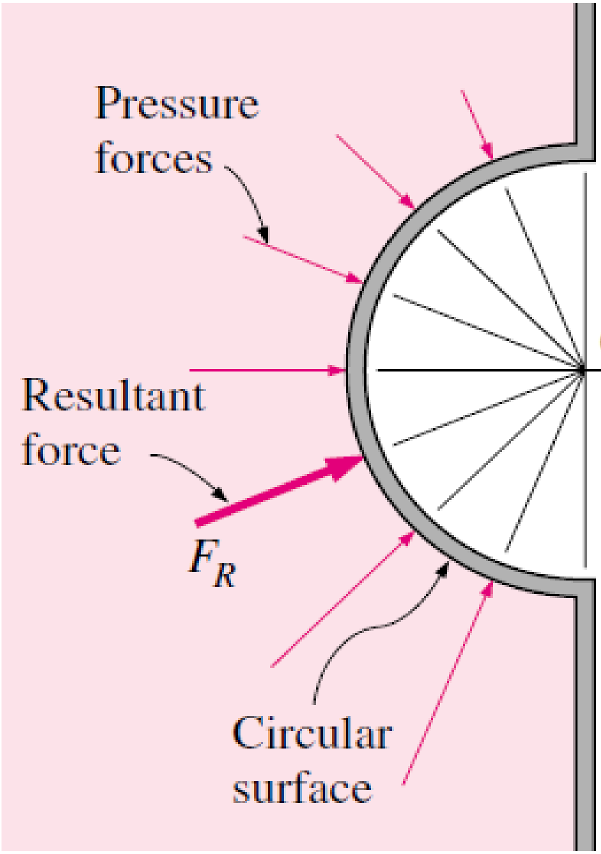
When a curved surface is above the liquid, the weight of the liquid and the vertical component of the hydrostatic force act in the opposite directions.

Hydrostatic Forces on Curved Surfaces

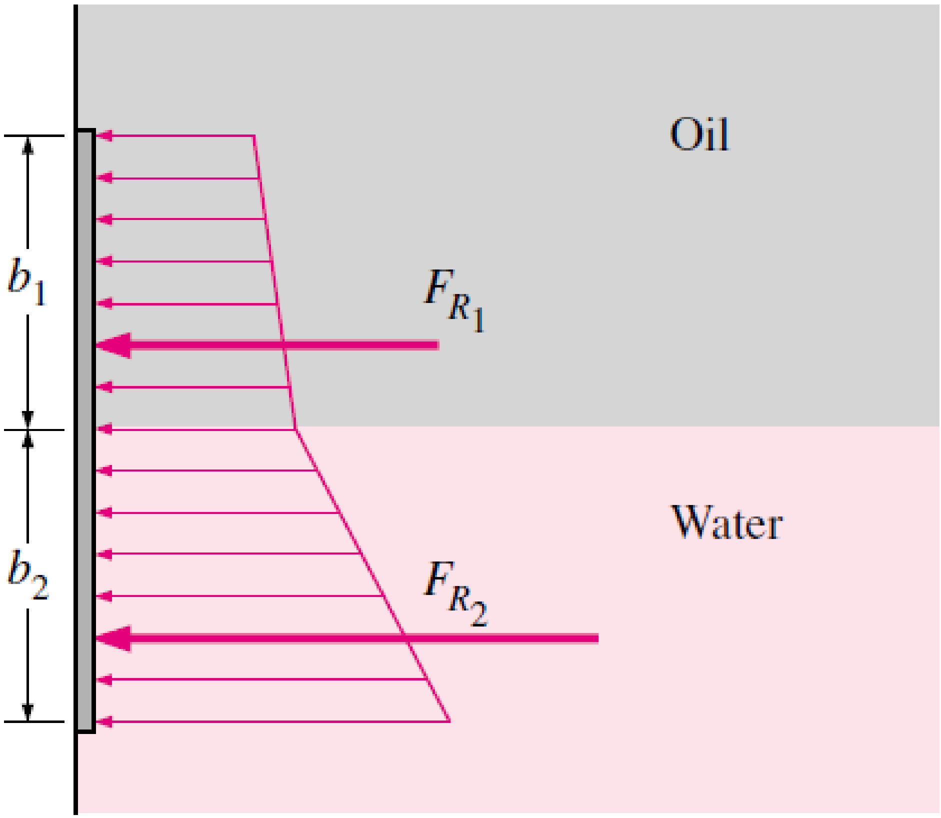
- Horizontal force component on curved surface: $F_H=F_x$. Line of action on vertical plane gives y coordinate of center of pressure on curved surface.

- Vertical force component on curved surface: $F_V=F_y+W$, where W is the weight of the liquid in the enclosed block $W=\rho gV$. x coordinate of the center of pressure is a combination of line of action on horizontal plane (centroid of area) and line of action through volume (centroid of volume).

- Magnitude of force $F_R=(F_H^2+F_V^2)^{1/2}$ & Angle of force is a = tan-1(F_V/F_H)



The hydrostatic force acting on a circular surface always passes through the center of the circle since the pressure forces are normal to the surface and they all pass through the center.



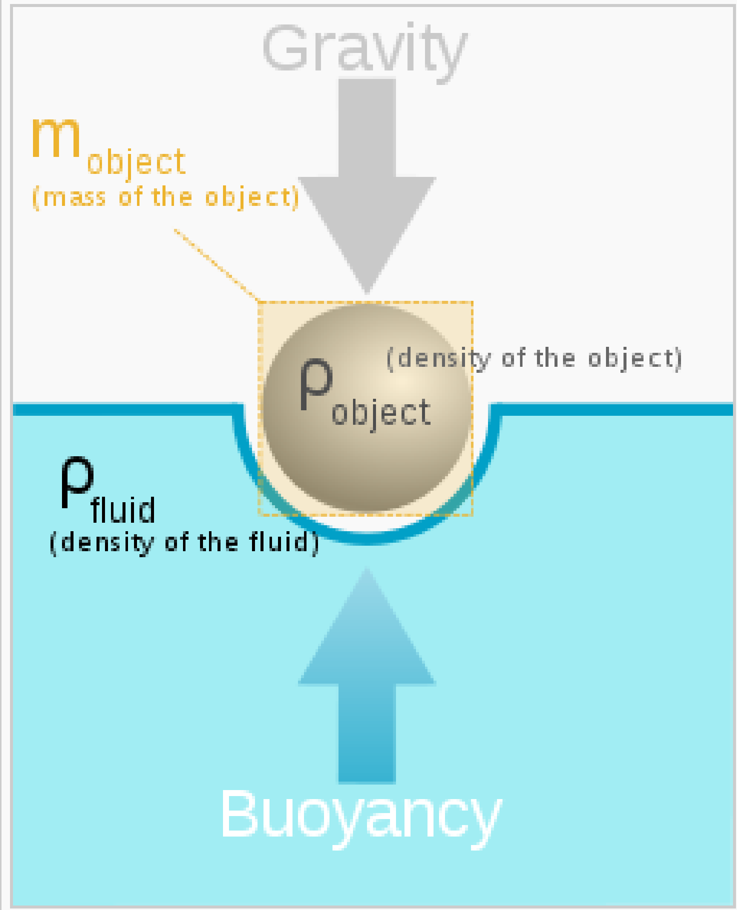
Hydrostatic force on a surface submerged in a multilayered fluid can be determined by considering parts of the surface in different fluids as different surfaces.

Buoyancy and Stability

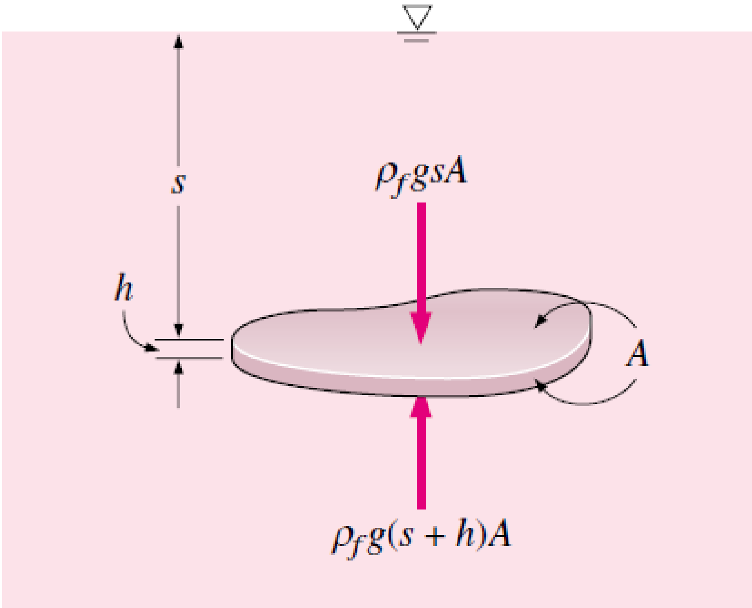
- Why objects feel lighter and weighs less in a liquid than it does in air?
- Objects made of wood or other lighter materials float on water. Why?

- Buoyant force F_B : The force offered by the fluid that tends to lift the body.
- Buoyant force is caused by the increase in pressure in a fluid with depth.

Buoyancy is due to the fluid displaced by a body.



The forces at work in buoyancy

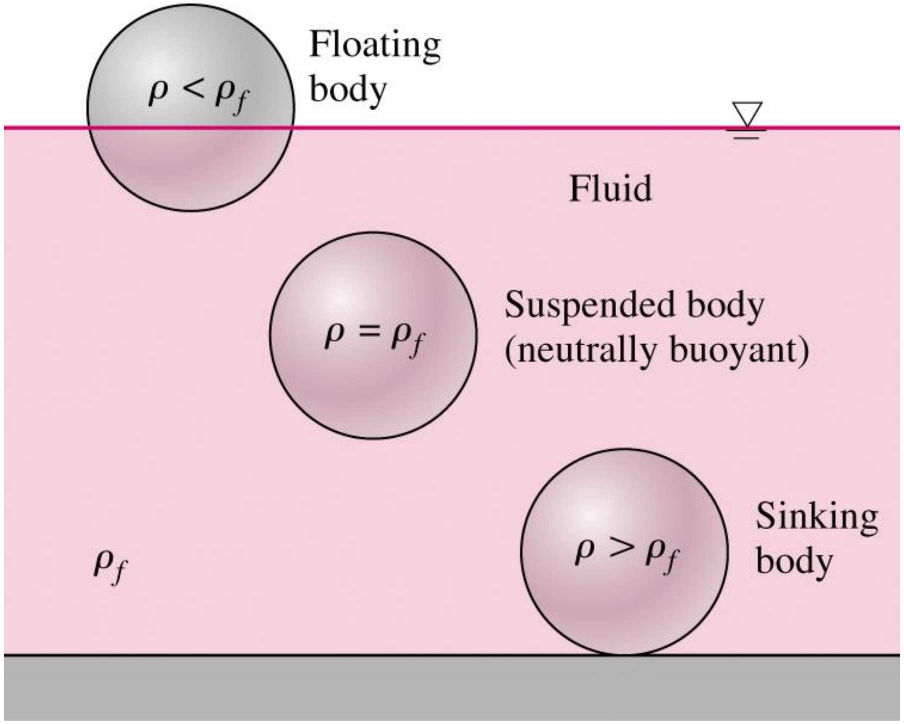


A flat plate of uniform thickness h submerged in a liquid parallel to the free surface

$$F_B = F_{\text{bottom}} - F_{\text{top}} = \rho_f g(s+h)A - \rho_f gsA = \rho_f ghA = \rho_f gV$$

Buoyant force acting on the plate is equal to the weight of the liquid displaced by the plate

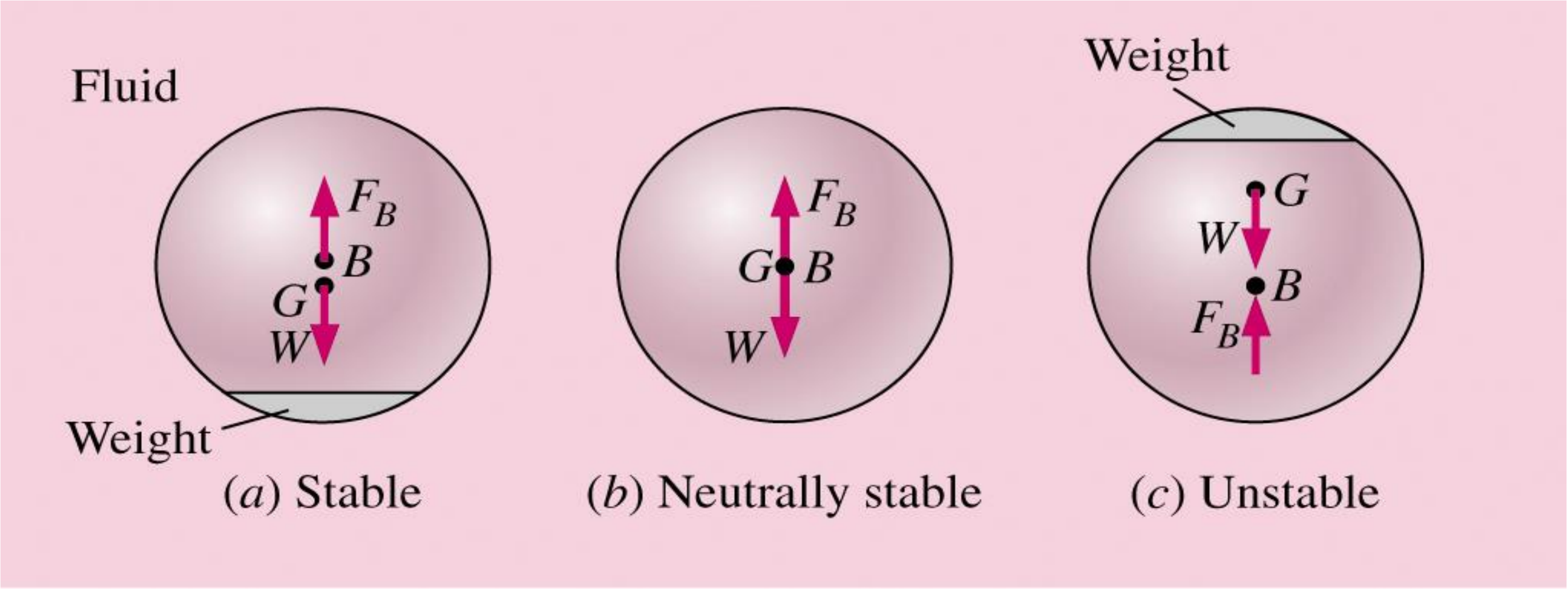
- It is independent of distance (s) of the body from the free surface
- It is independent of the density of the solid body



$$F_B = W \quad \rightarrow \quad \rho_f g\, V_{sub} = \rho_{ave,\, body}\, g V_{total} \quad \rightarrow \quad \frac{V_{sub}}{V_{total}} = \frac{\rho_{ave,\, body}}{\rho_f}$$

For Floating bodies , the weight of the entire body must be equal to buoyant force, which is the weight of the fluid whose volume is equal to the volume of the submerged portion of the floating body.

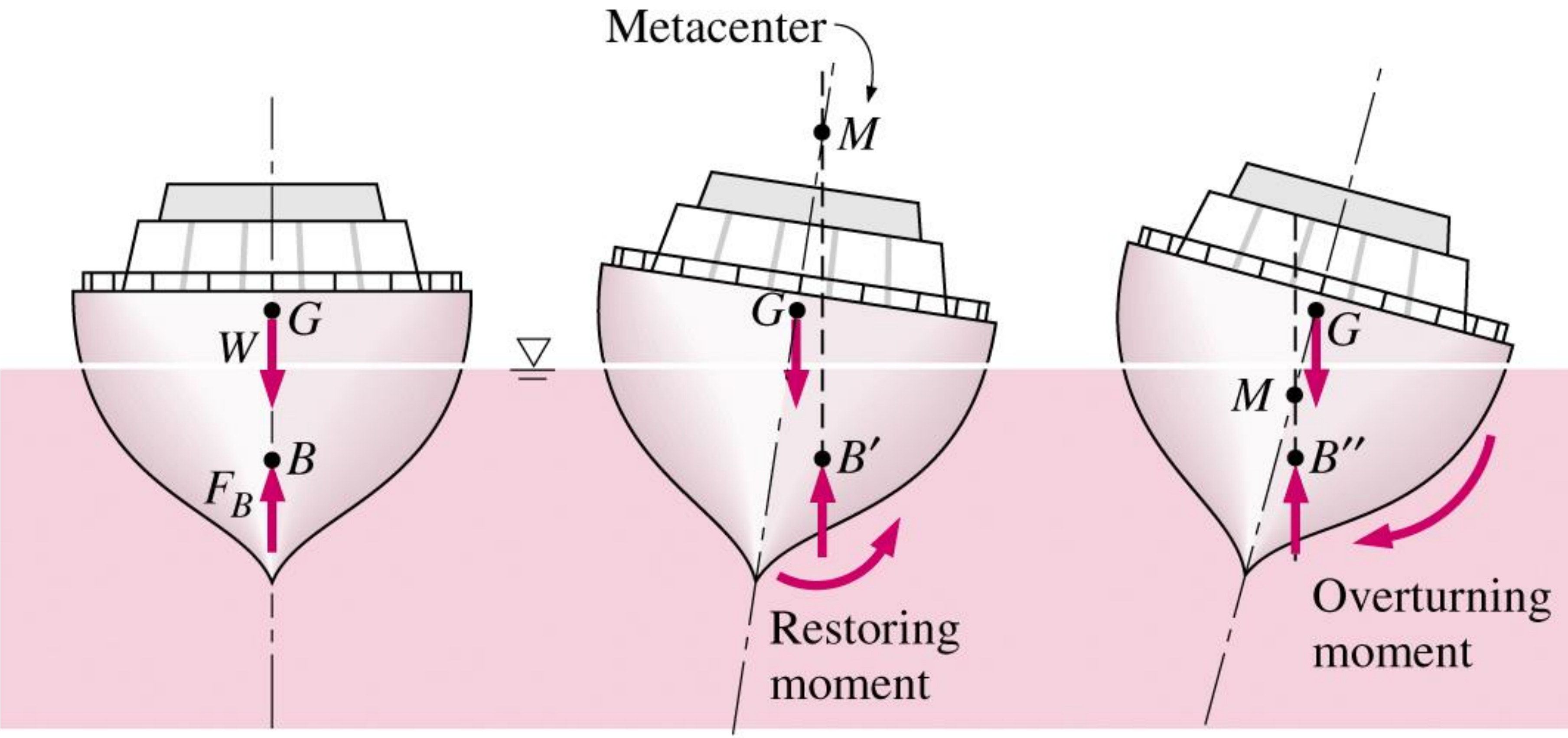
Stability of Immersed Bodies



- Rotational stability of immersed bodies depends upon relative location of center of gravity G and center of buoyancy B.

  - *G* below *B* : stable
  - *G* above *B* : unstable
  - *G* coincides with *B* : neutrally stable.

Stability of Floating Bodies



(a) Stable (b) Stable (c) Unstable

Fluid Kinematics

Chapter 04

Introduction

- Kinematics means the study of motion without considering the forces and moments that cause the motion.

- Kinematics involves position, velocity, and acceleration, not force.

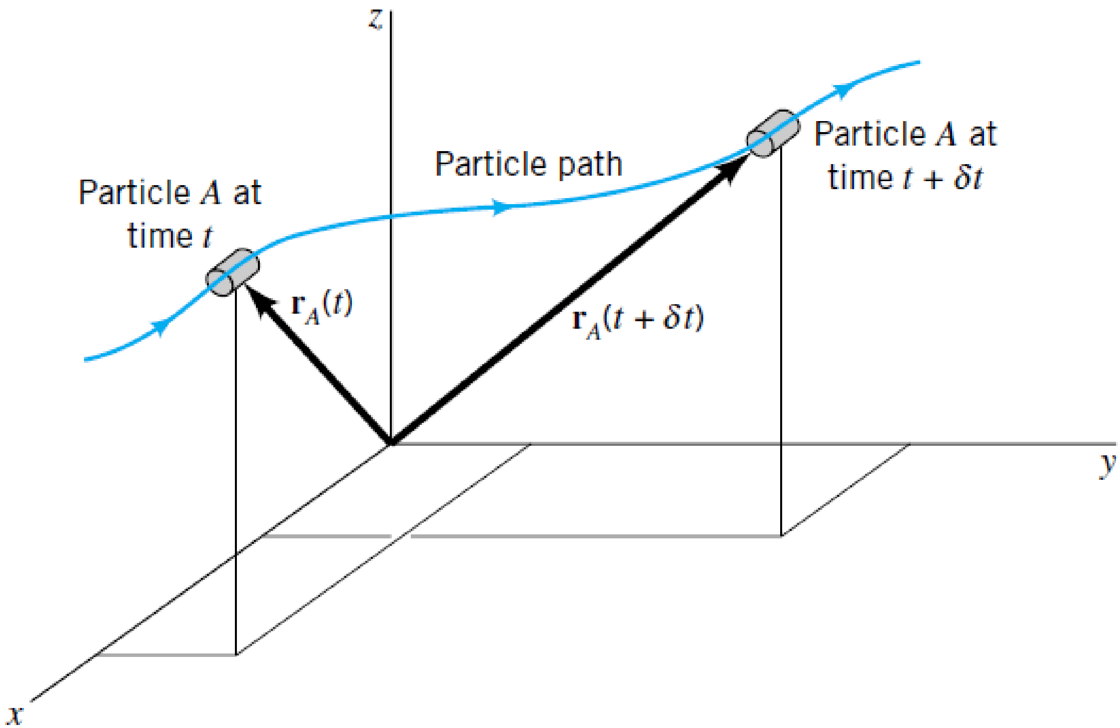
- Fluid kinematics is the study of how fluids flow and how to describe fluid motion without considering the forces and moments that cause the motion.

- Fluid kinematics describing how a fluid particle translates, distorts, and rotates, and how to visualize flow fields.

- Topics

  - Scalar and Vector Fields, Flow Field
  - Descriptions of fluid flow.
  - Material Derivative or Substantial Derivative
  - Fundamentals of Flow visualization.
  - Plots of fluid flow data.
  - Fundamental kinematic properties of fluid motion and deformation.

Field Representation



Particle locations in terms of its position vector

The representation of fluid parameters as functions of the spatial and temporal coordinates is termed a field representation of the flow

Scalar and Vector Fields

- Scalar: Scalar is a quantity which can be expressed by a single number representing its magnitude.

  Example: mass, density and temperature.

  - Scalar Field :If at every point in a region, a scalar function has a defined value, the region is called a scalar field.

    Example: Temperature distribution in a rod.

- Vector: Vector is a quantity which is specified by both magnitude and direction.

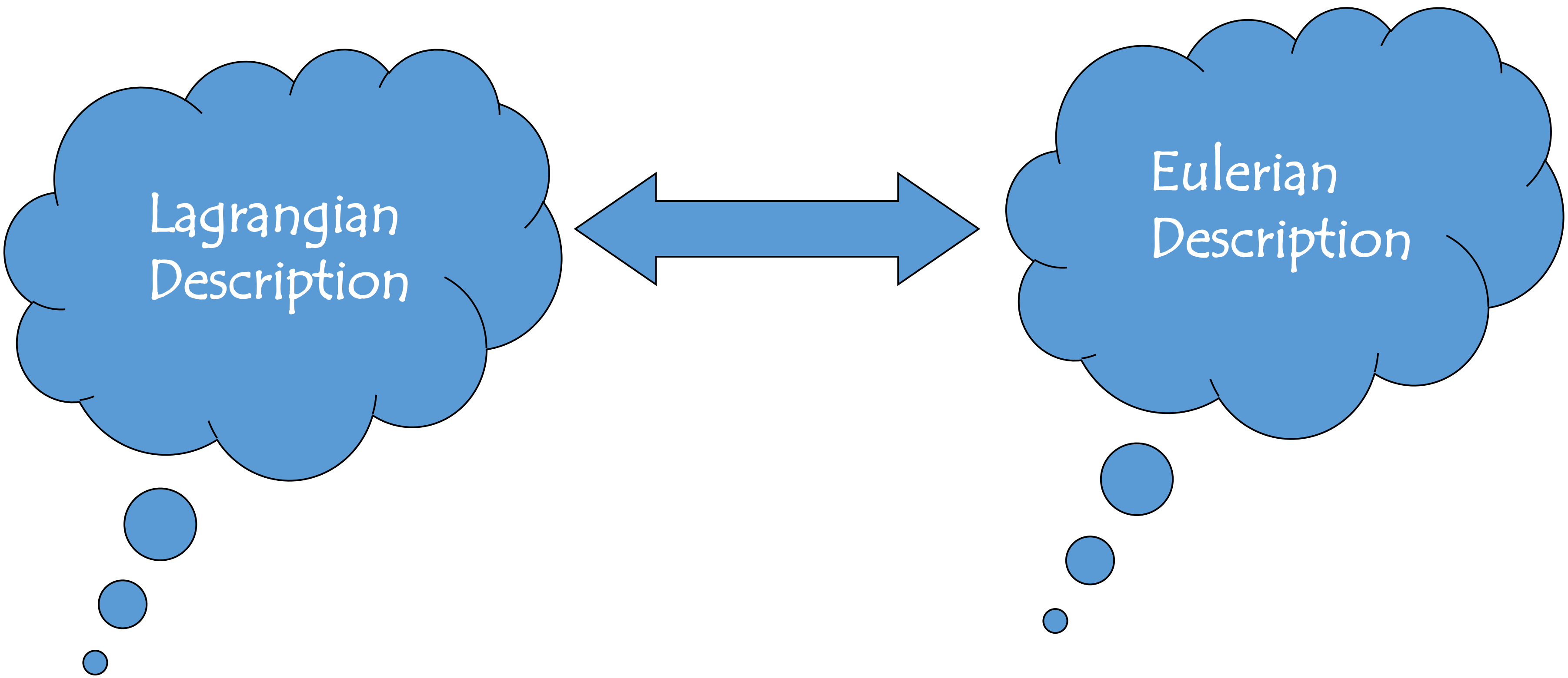
  Example: Force, Velocity and Displacement.

  - Vector Field :If at every point in a region, a vector function has a defined value, the region is called a vector field.

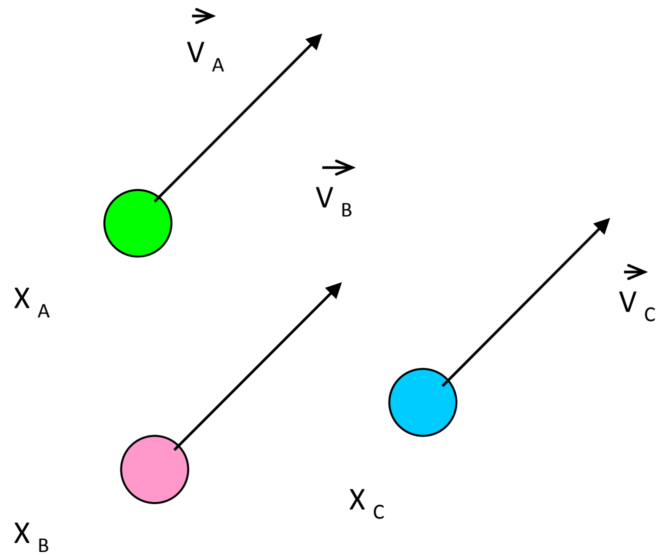
    Example: velocity field of a flowing fluid .

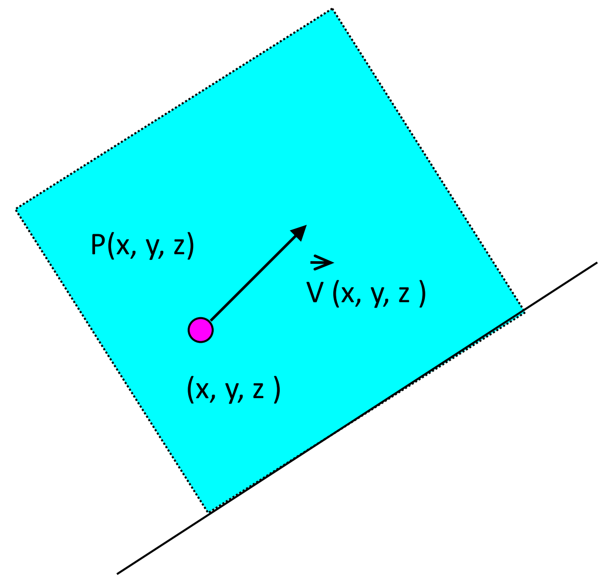
Descriptions of Fluid Flow

- There are two general approaches in analyzing fluid mechanics problems



Lagrangian Vs Eulerian





In the lagrangian description, one must keep track of the position and velocity of individual particles

In the Eulerian description, one defines field variables, such as the pressure field and the velocity field at any location and instant in time

Lagrangian Description

- Lagrangian description of fluid flow tracks the position and velocity of individual particles. (eg. Brilliard ball on a pooltable.)

- Motion is described based upon Newton's laws of motion.

- Difficult to use for practical flow analysis.
  - Fluids are composed of billions of molecules.
  - Interaction between molecules hard to describe /model.

- However, useful for specialized applications
  - Sprays, particles, bubble dynamics, rarefied gases.
  - Coupled Eulerian-Lagrangian methods.

- Named after Italian mathematician Joseph Louis Lagrange (1736-1813).

Eulerian Description

- Eulerian description of fluid flow: a flow domain or control volume is defined by which fluid flows in and out.

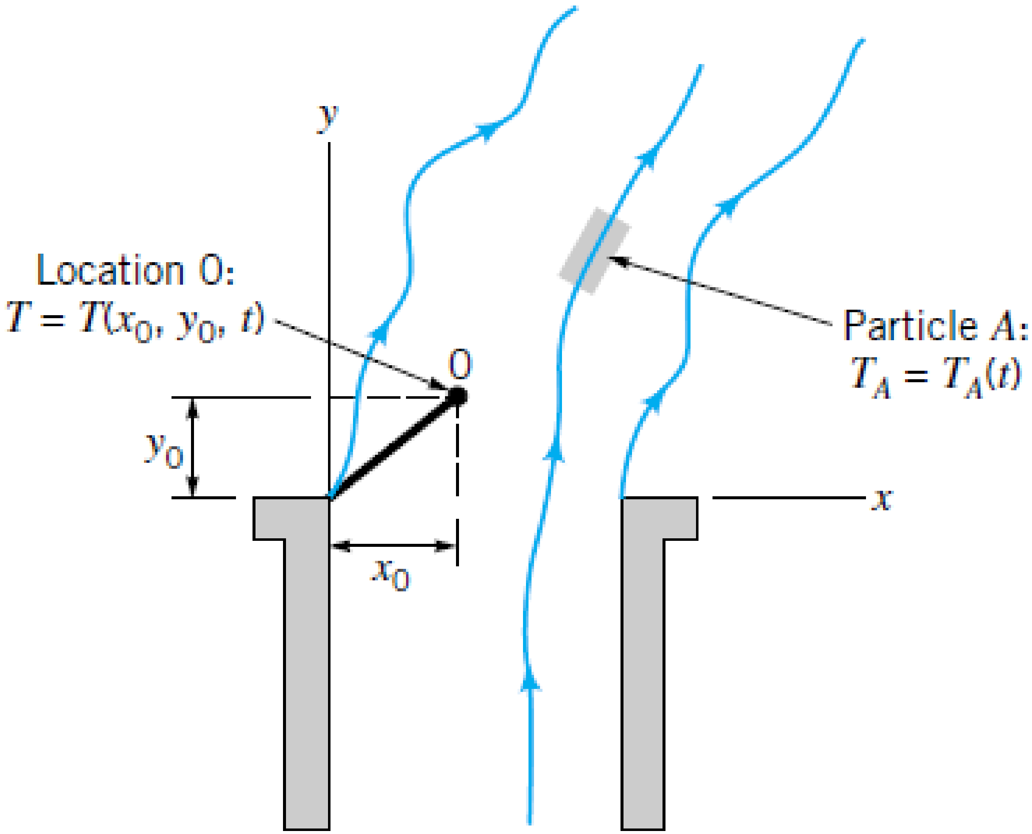
- We define field variables which are functions of space and time.
  - Pressure field, P=P(x,y,z,t)
  - Velocity field, $\vec{V} = \vec{V}(x,y,z,t)$ $\quad \vec{V} = u(x,y,z,t)\vec{i} + v(x,y,z,t)\vec{j} + w(x,y,z,t)\vec{k}$
  - Acceleration field, $\vec{a} = \vec{a}(x,y,z,t)$ $\quad \vec{a} = a_x(x,y,z,t)\vec{i} + a_y(x,y,z,t)\vec{j} + a_z(x,y,z,t)\vec{k}$

  These (and other) field variables define the flow field.

- Well suited for formulation of initial boundary-value problems (PDE's).

- Named after Swiss mathematician Leonhard Euler (1707-1783).

Eulerian and Lagrangian descriptions of temperature of a flowing fluid.



Eulerian Description

In the Eulerian method one may attach a temperature-measuring device to the top of the chimney (point O) and record the temperature at that point as a function of time. At different times there are different fluid particles passing by the stationary device. Thus, one would obtain the temperature, *T*, for that location ($x = x_o$, $y=y_o$, $z= z_o$) as a function of time. That is, $T = T(x_o, y_o, z_o, t)$

Lagrangian Description

In the Lagrangian method, one would attach the temperature-measuring device to a particular fluid particle (particle A) and record that particle's temperature as it moves about. Thus, one would obtain that particle's temperature as a function of time,

$$T_A = T_A(t)$$

Acceleration Field

- Consider a fluid particle and Newton's second law,

$$\vec{F}_{particle} = m_{particle}\vec{a}_{particle}$$

- The acceleration of the particle is the time derivative of the particle's velocity.

$$\vec{a}_{particle} = \frac{d\vec{V}_{particle}}{dt}$$

$$\vec{V}_{particle} = \vec{V}\left(x_{particle}(t), y_{particle}(t), z_{particle}(t)\right)$$

- However, particle velocity at a point is the same as the fluid velocity,
- To take the time derivative of, chain rule must be used.

$$\vec{a}_{particle} = \frac{\partial \vec{V}}{\partial t}\frac{dt}{dt} + \frac{\partial \vec{V}}{\partial x}\frac{dx_{particle}}{dt} + \frac{\partial \vec{V}}{\partial y}\frac{dy_{particle}}{dt} + \frac{\partial \vec{V}}{\partial z}\frac{dz_{particle}}{dt}$$

- Since $$\frac{dx_{particle}}{dt}=u,\frac{dy_{particle}}{dt}=v,\frac{dz_{particle}}{dt}=w$$

$$\vec{a}_{particle}=\frac{\partial\vec{V}}{\partial t}+u\frac{\partial\vec{V}}{\partial x}+v\frac{\partial\vec{V}}{\partial y}+w\frac{\partial\vec{V}}{\partial z}$$

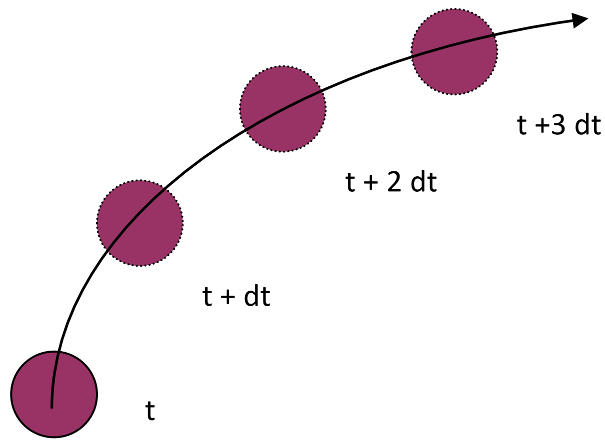
- In vector form, the acceleration can be written as

$$\vec{a}(x,y,z,t)=\frac{d\vec{V}}{dt}=\frac{\partial\vec{V}}{\partial t}+(\vec{V}.\vec{\nabla})\vec{V}$$

- First term is called the local acceleration and is nonzero only for unsteady flows.

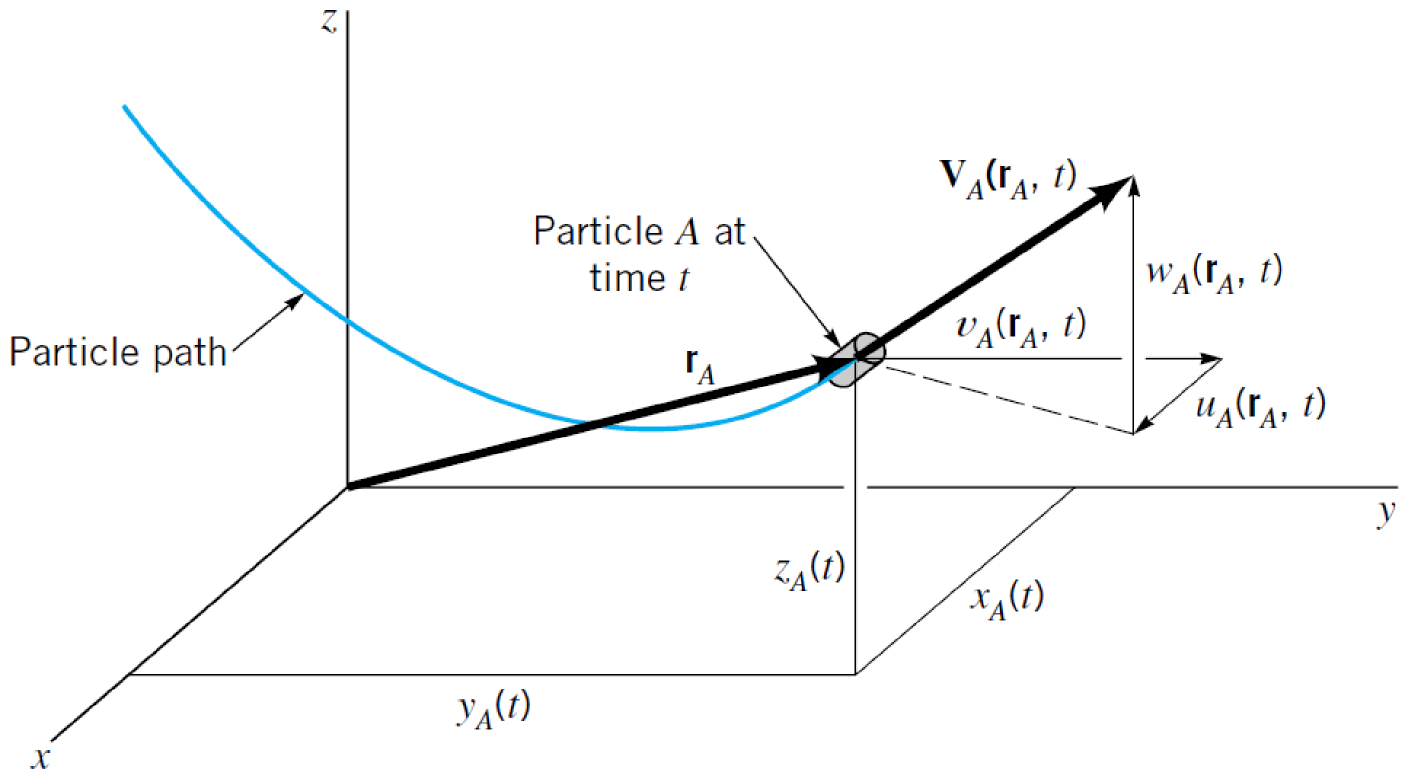
- Second term is called the advective or convective acceleration and accounts for the effect of the fluid particle moving to a new location in the flow, where the velocity is different.

Material Derivative



The material derivative D/Dt is defined by following a fluid particle as it moves throughout the flow field.

Material Derivative



$$\mathbf{V}_A = \mathbf{V}_A(\mathbf{r}_A, t) = \mathbf{V}_A[x_A(t), y_A(t), z_A(t), t]$$

$$\mathbf{a}_A(t) = \frac{d\mathbf{V}_A}{dt} = \frac{\partial \mathbf{V}_A}{\partial t} + \frac{\partial \mathbf{V}_A}{\partial x}\frac{dx_A}{dt} + \frac{\partial \mathbf{V}_A}{\partial y}\frac{dy_A}{dt} + \frac{\partial \mathbf{V}_A}{\partial z}\frac{dz_A}{dt}$$

$$\mathbf{a}_A = \frac{\partial \mathbf{V}_A}{\partial t} + u_A \frac{\partial \mathbf{V}_A}{\partial x} + v_A \frac{\partial \mathbf{V}_A}{\partial y} + w_A \frac{\partial \mathbf{V}_A}{\partial z}$$

the above equation is valid for any particle, we can drop the reference to particle A and obtain the acceleration field from the velocity field as

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u \frac{\partial \mathbf{V}}{\partial x} + v \frac{\partial \mathbf{V}}{\partial y} + w \frac{\partial \mathbf{V}}{\partial z}$$

$$a_x = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$

$$a_y = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}$$

$$a_z = \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z}$$

The above result is often written in shorthand notation as

$$\mathbf{a} = \frac{D\mathbf{V}}{Dt}$$

where the operator

$$\frac{D(\ )}{Dt} \equiv \frac{\partial(\ )}{\partial t} + u\frac{\partial(\ )}{\partial x} + v\frac{\partial(\ )}{\partial y} + w\frac{\partial(\ )}{\partial z}$$

is termed the material derivative or substantial derivative

An often-used shorthand notation for the material derivative operator is

$$\frac{D(\ )}{Dt} = \frac{\partial(\ )}{\partial t} + (\mathbf{V} \cdot \nabla)(\ )$$

the rate of change of temperature as

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T$$

Remarks about Material Derivative

- The total derivative operator d/dt is called the material derivative and is often given special notation, D/Dt.

- Advective acceleration is nonlinear. It is the source of many phenomenon and primary challenge in solving fluid flow problems.

- Provides transformation between Lagrangian and Eulerian frames.

- Other names for the material derivative include: total, particle, Lagrangian, Eulerian, and substantial derivative.

Flow Visualization

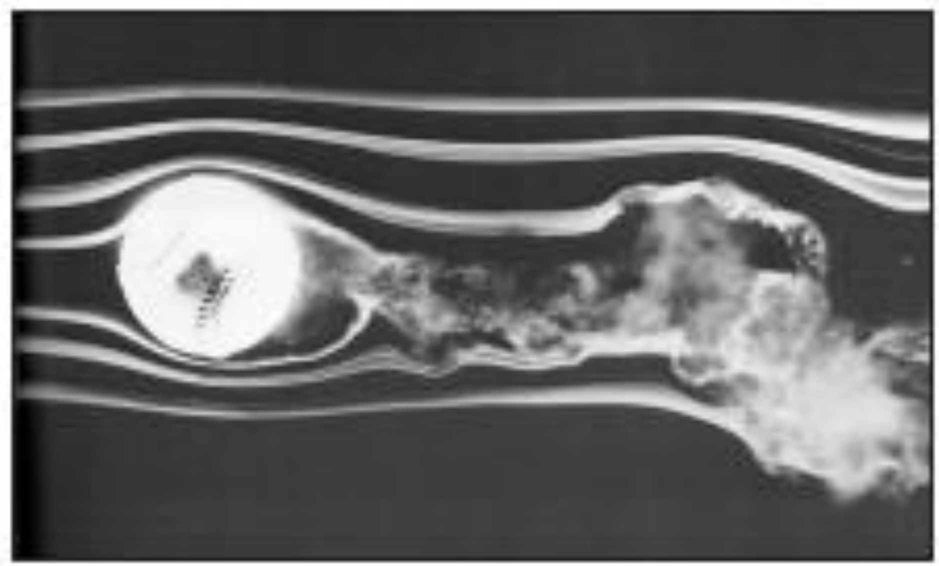
Flow visualization is the visual examination of flow-field features.

Important for both physical experiments and numerical (CFD) solutions.

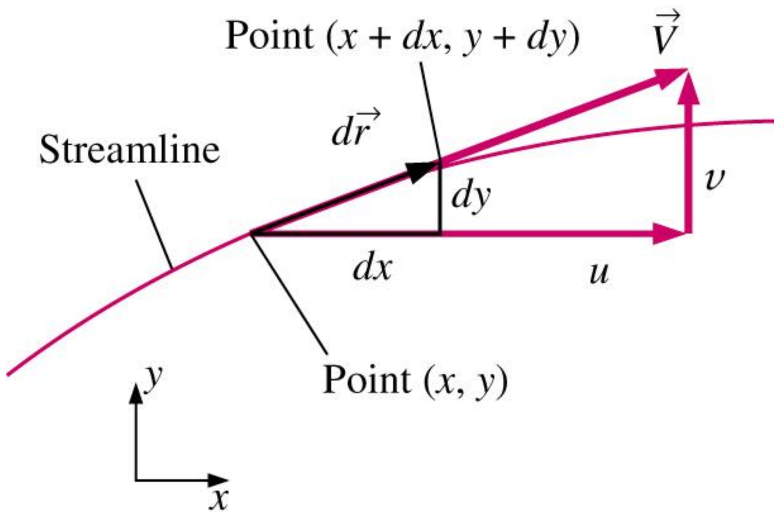
While quantitative study of fluid dynamics requires advanced mathematics, much can be learned from flow visualization

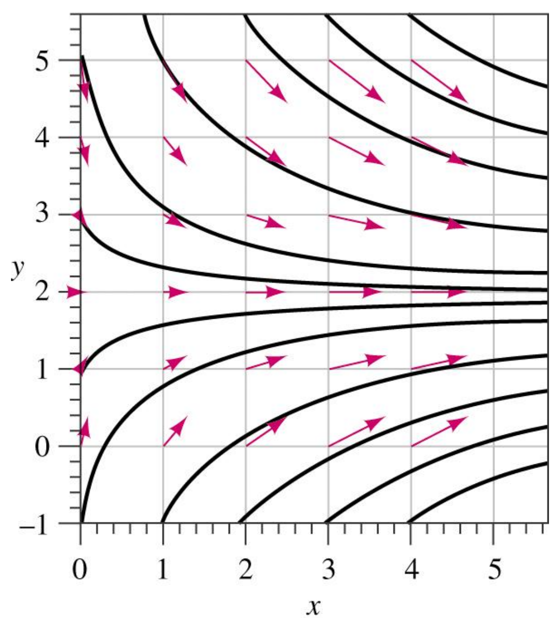
Numerous methods

- Streamlines
- Pathlines
- Streaklines
- Refractive techniques
- Surface flow techniques



Streamlines





- A Streamline is a curve that is everywhere tangent to the instantaneous local velocity vector.
- Consider an arc length

$$d\vec{r} = dx\vec{i} + dy\vec{j} + dz\vec{k}$$

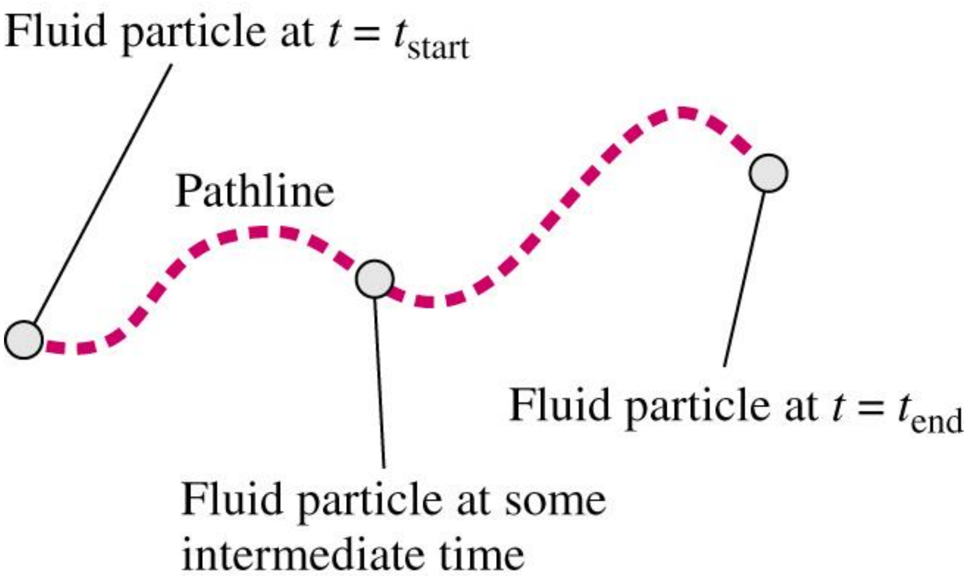
$d\vec{r}$ must be parallel to the local velocity vector

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

- Geometric arguments results in the equation for a streamline

$$\frac{dr}{V} = \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

Pathlines

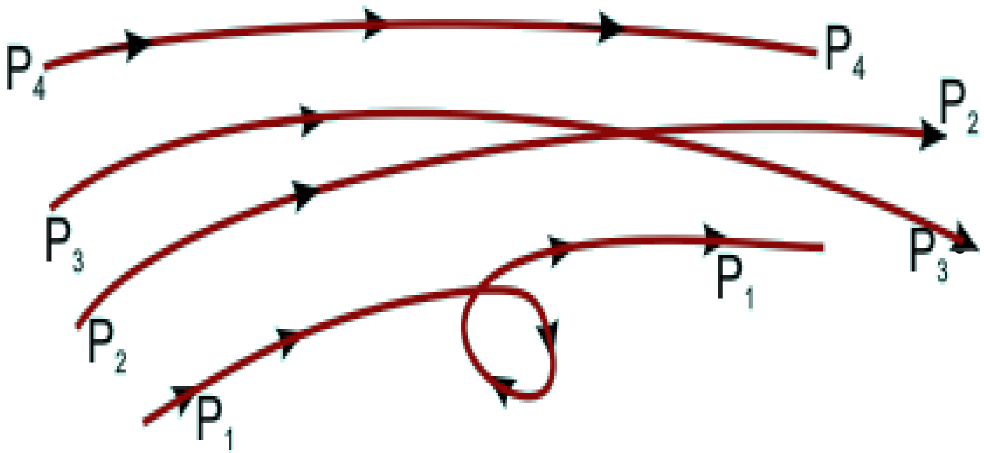


- A Pathline is the actual path traveled by an individual fluid particle over some time period.
- Same as the fluid particle's material position vector

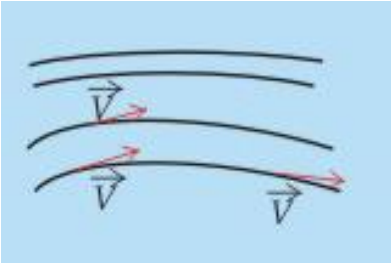
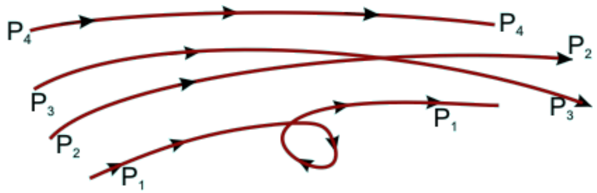
$$\left(x_{particle}(t), y_{particle}(t), z_{particle}(t)\right)$$

- Particle location at time t:

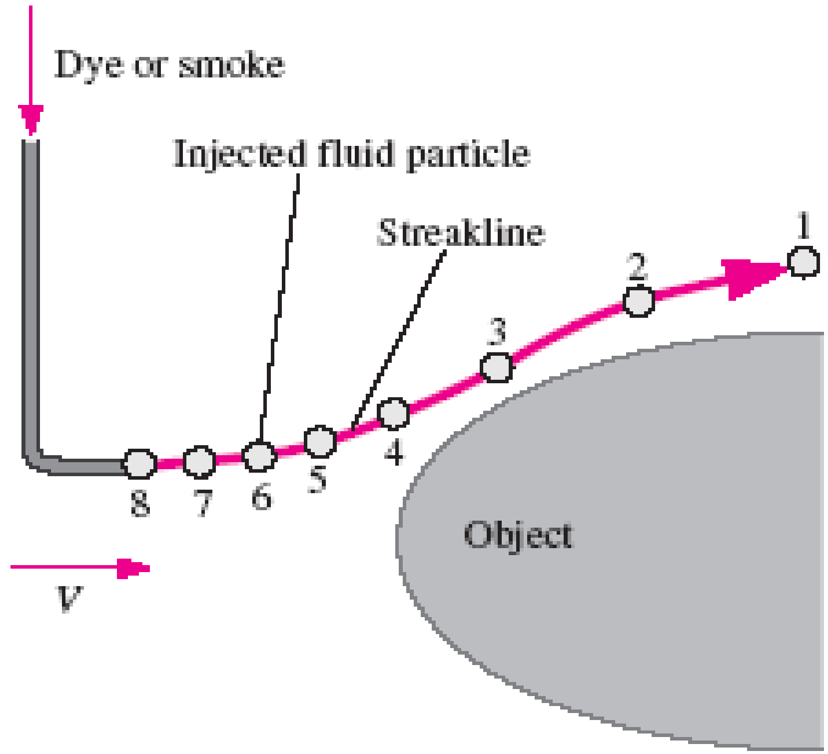
$$\vec{x} = \vec{x}_{start} + \int_{t_{start}}^{t} \vec{V} dt$$



- Particle Image Velocimetry (PIV) is a modern experimental technique to measure velocity field over a plane in the flow field.

Stream Line	Path Line
	
This is an imaginary curve in a flow field for a fixed instant of time, tangent to which gives the instantaneous velocity at that point .	This refers to a path followed by a fluid particle over a period of time.
Two stream lines can never intersect each other, as the instantaneous velocity vector at any given point is unique.	Two path lines can intersect each other as or a single path line can form a loop as different particles or even same particle can arrive at the same point at different instants of time.

Streaklines



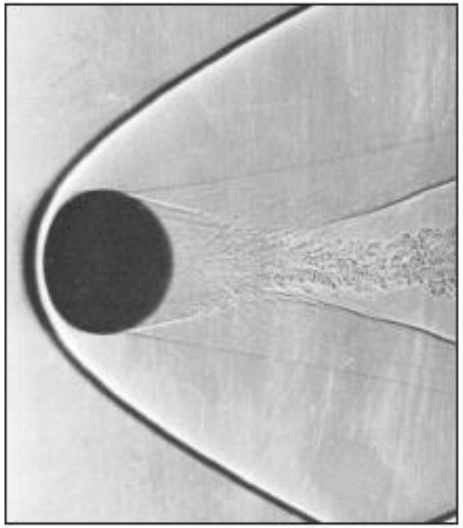
A streak line is the locus of the temporary locations of all particles that have passed though a fixed point in the flow field at any instant of time

Easy to generate in experiments: dye in a water flow, or smoke in an airflow.

$$\vec{x} = \vec{x}_{injection} + \int_{t_{inject}}^{t_{present}} \vec{V} dt$$

Refractive Flow Visualization Techniques

- Based on the refractive property of light waves in fluids with different index of refraction, one can visualize the flow field: shadowgraph technique and schlieren technique.



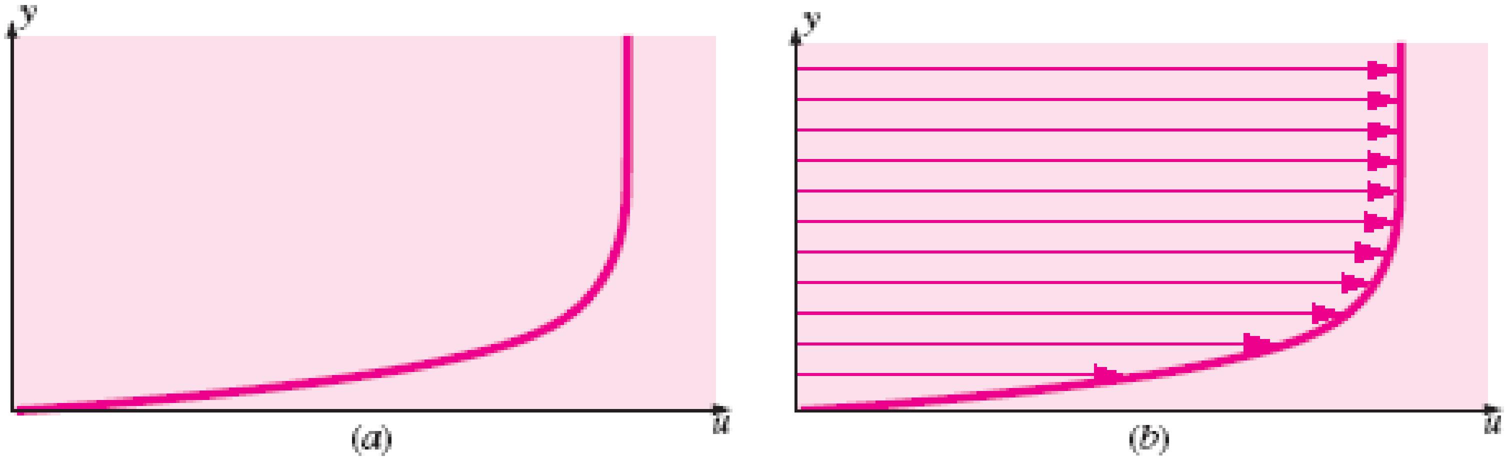
Plots of Data

- A Profile plot indicates how the value of a scalar property varies along some desired direction in the flow field.

- A Vector plot is an array of arrows indicating the magnitude and direction of a vector property at an instant in time.

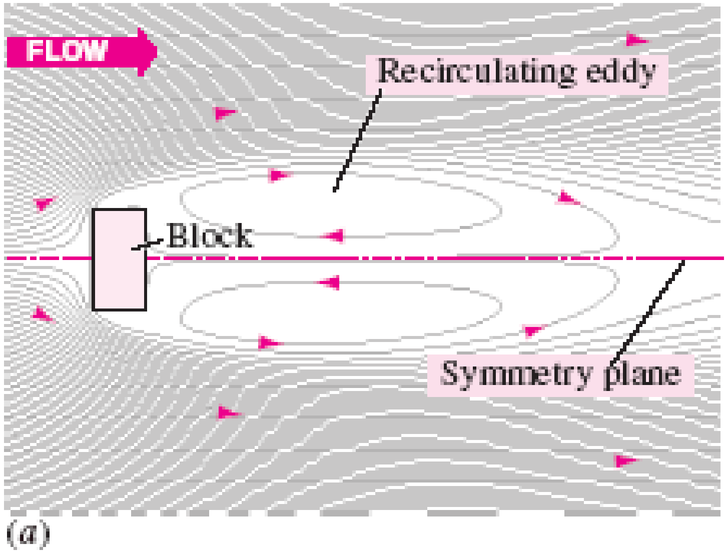
- A Contour plot shows curves of constant values of a scalar property (or magnitude of a vector property) at an instant in time.

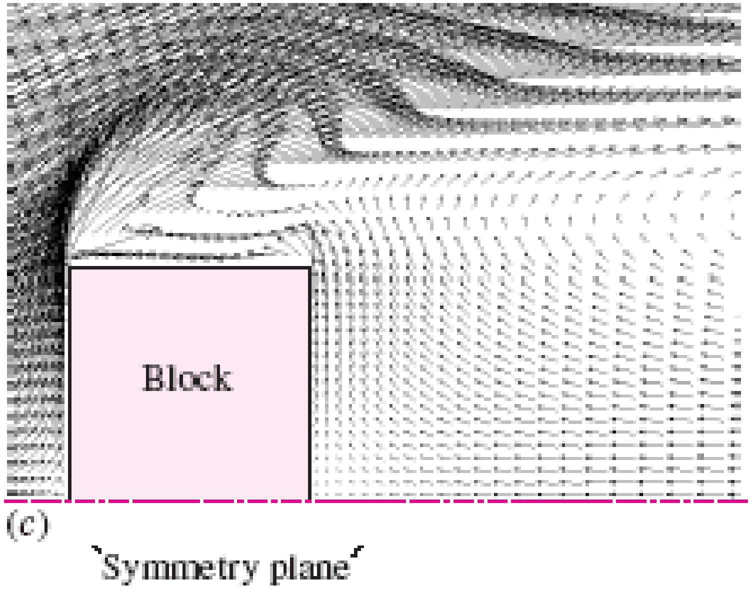
Profile plot



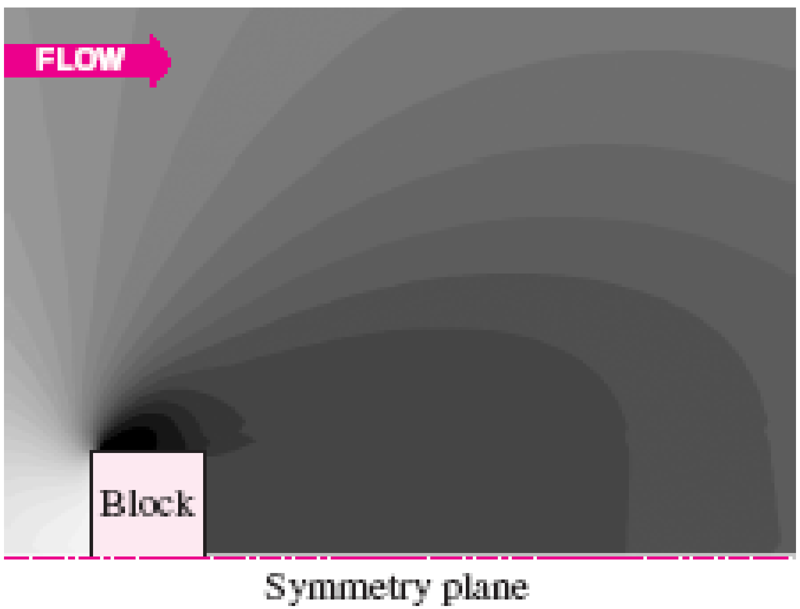
- Profile plots of the horizontal component of velocity as a function of vertical distance; flow in the boundary layer growing along a horizontal flat plate.

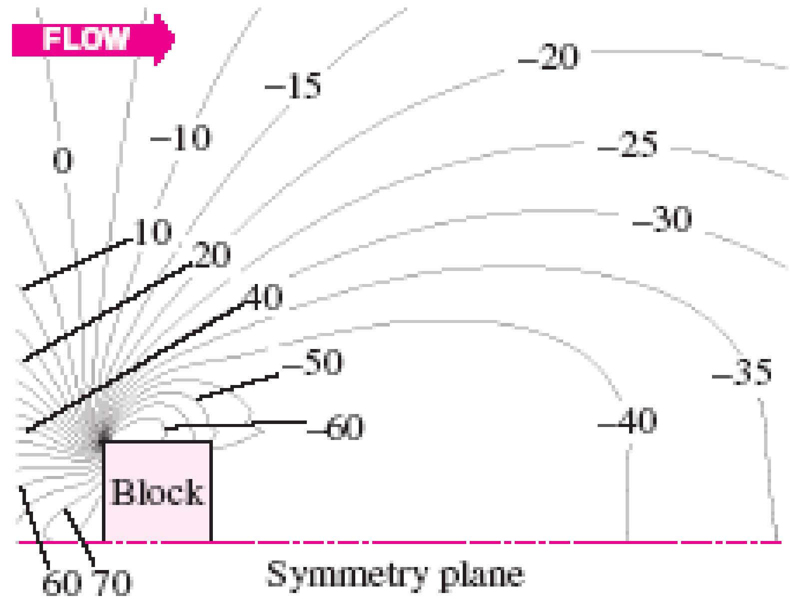
Vector plot


FLOW
Recirculating eddy
Block
Symmetry plane
(a)

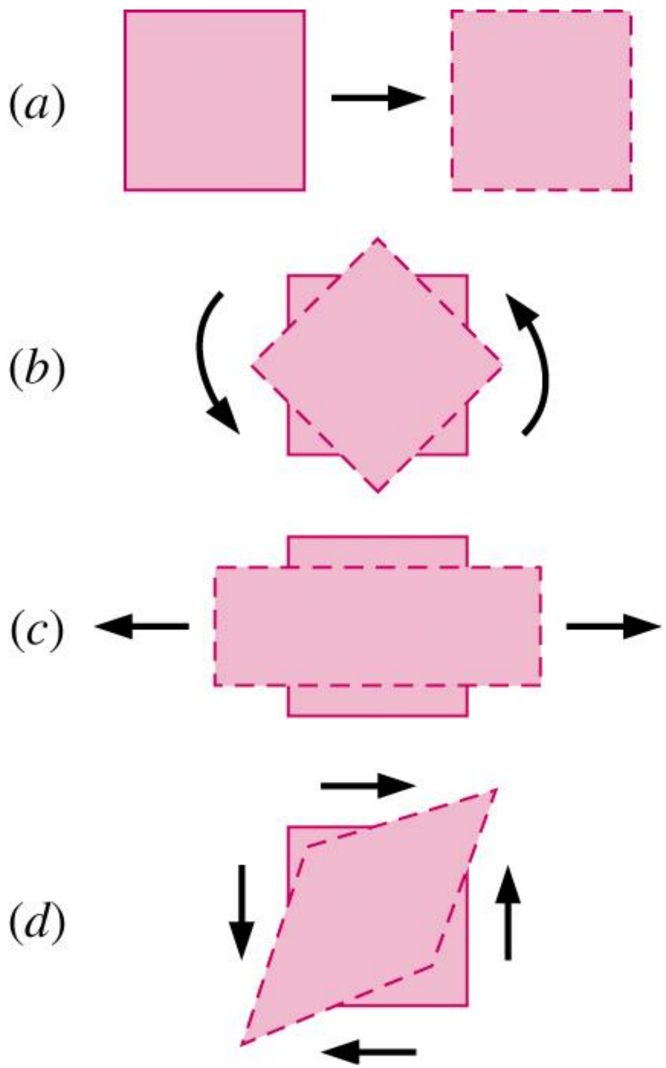

Block
(c)
Symmetry plane

Contour plot





Kinematic Description



- In fluid mechanics, an element may undergo four fundamental types of motion.
  - Translation
  - Rotation
  - Linear strain
  - Shear strain
- Because fluids are in constant motion, motion and deformation are described in terms of rates
  - velocity: rate of translation
  - angular velocity: rate of rotation
  - linear strain : rate of linear strain
  - shear strain : rate of shear strain

Rate of Translation

- To be useful, these rates must be expressed in terms of velocity and derivatives of velocity
- The rate of translation vector is described as the velocity vector. In Cartesian coordinates:

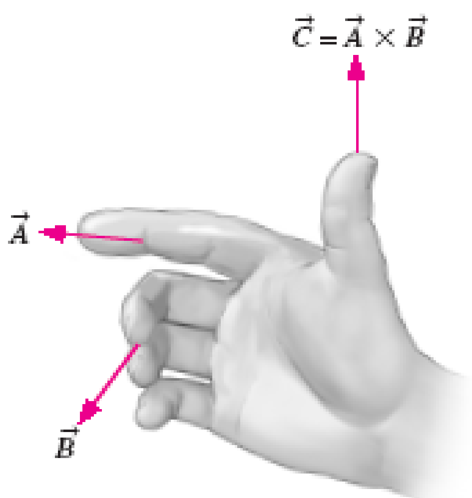
$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

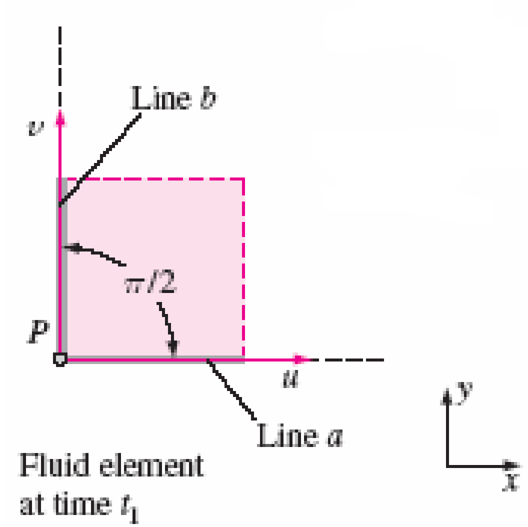
Rate of Rotation

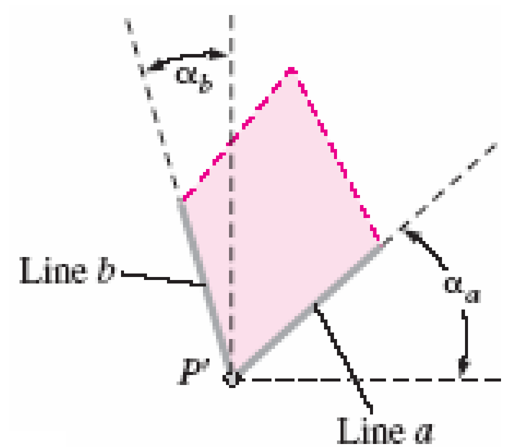
- Rate of rotation or angular velocity at a point in the xy plane is equal to the time derivative of the average rotation angle.

The rate of rotation vector in Cartesian coordinates:

$$\vec{\omega} = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} + \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\vec{j} + \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$$



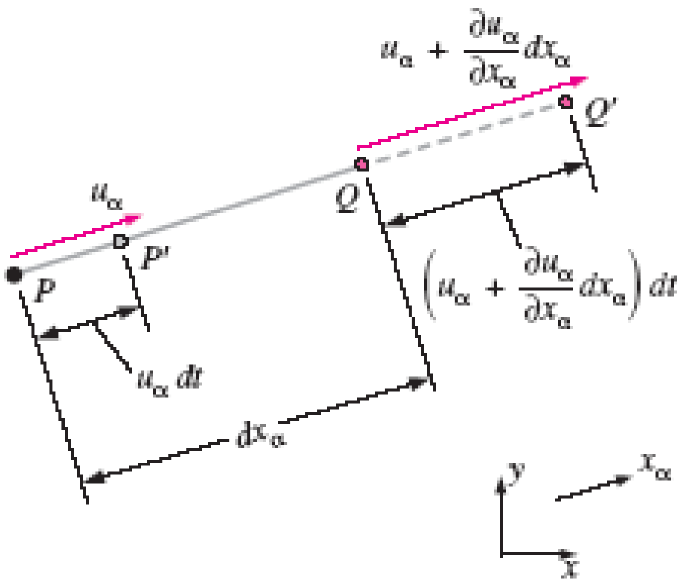




Linear Strain Rate

- Linear Strain Rate is defined as the rate of increase in length per unit length.
- Linear strain rate in Cartesian coordinates

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}, \varepsilon_{yy} = \frac{\partial v}{\partial y}, \varepsilon_{zz} = \frac{\partial w}{\partial z}$$

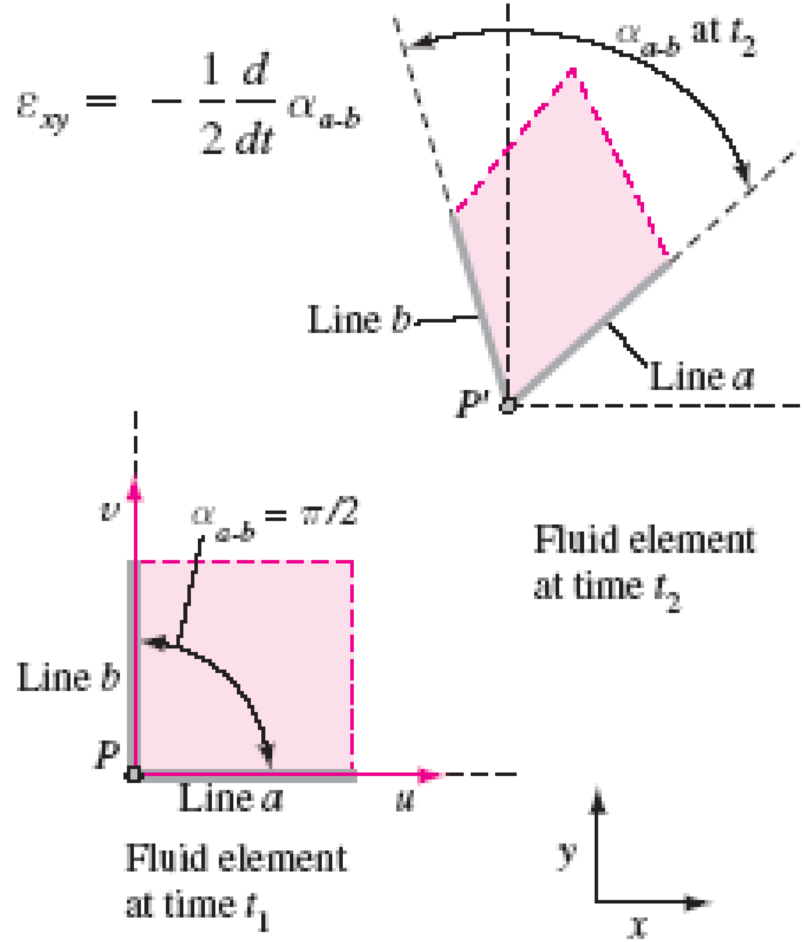


- Volumetric strain rate in Cartesian coordinates

$$\frac{1}{V}\frac{DV}{Dt} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

Shear Strain Rate

- Shear Strain Rate at a point is defined as half of the rate of decrease of the angle between two initially perpendicular lines that intersect at a point.
- Shear strain rate can be expressed in Cartesian coordinates as:



$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \varepsilon_{zx} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right), \varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

Strain - rate Tensor

We can combine linear strain rate and shear strain rate into one symmetric second-order tensor called the strain-rate tensor.

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix} = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) & \frac{\partial v}{\partial y} & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) & \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) & \frac{\partial w}{\partial z} \end{pmatrix}$$

Circulation and Vorticity

- Circulation :

It is defined mathematically as the line integral of the tangential velocity about a closed path (contour)

$$\Gamma = \oint V \cos\theta . ds$$

V - velocity in the flow field at the element ds

θ - angle between V and tangent to the path (in the positive anticlockwise direction along the path) at the point

- Vorticity (ζ or Ω) :

It is also defined as circulation per unit of enclosed area.

It is a measure of rotation of a fluid particle equal to twice the angular velocity of the fluid particle.

Vorticity and Rotationality

- The vorticity vector is defined as the curl of the velocity vector $\vec{\zeta} = \vec{\nabla} \times \vec{V}$

- Vorticity is equal to twice the angular velocity of a fluid particle. $\vec{\zeta} = 2\vec{\omega}$

- Cartesian coordinates

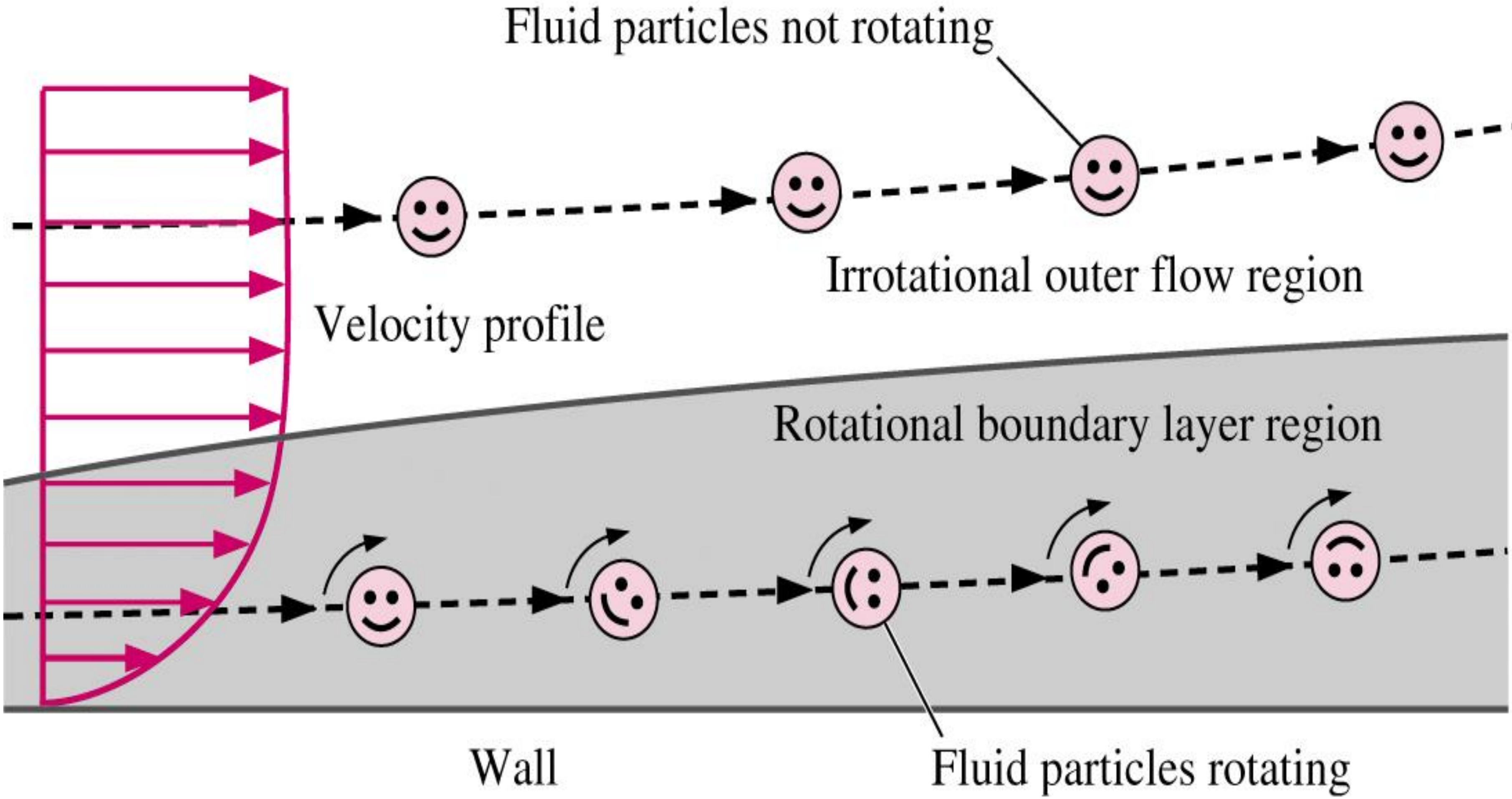
$$\vec{\zeta} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right)\vec{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right)\vec{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)\vec{k}$$

- Cylindrical coordinates

$$\vec{\zeta} = \left(\frac{1}{r}\frac{\partial u_z}{\partial \theta} - \frac{\partial u_\theta}{\partial z} \right)\vec{e}_r + \left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r} \right)\vec{e}_\theta + \left(\frac{\partial (r u_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta} \right)\vec{e}_z$$

- In regions where z = 0, the flow is called irrotational.
- Elsewhere, the flow is called rotational.

Rotational and Irrotational flow



- Fluid particles within viscous boundary layer near the solid wall are rotational.

- Fluid particles outside the boundary layer are irrotational.

- Rotation of fluid elements is associated with wakes, boundary layers, flow through turbomachinery and flow with heat transfer.

- When torque is applied to the fluid particle it will give rise to rotation; the torque is due to shear stress.

- The shear stress in turn dependent upon the viscosity, rotational flow occurs where the viscosity effect are predominant.

- In case were viscosity effects are small it can be assume as irrotational flow

The Stream Function

- Why do this?
  - Single variable ψ replaces (u,v). Once ψ is known, (u,v) can be computed.
  - Physical significance
    - Curves of constant ψ are streamlines of the flow
    - Difference in ψ between streamlines is equal to volume flow rate between streamlines
    - It can also be defined as the flux or flow rate between two streamlines. The unit of ψ is m^3/s (discharge per unit thickness of flow).
    - Existence of ψ means a possible case of fluid flow

The Stream Function

- Consider the continuity equation for an incompressible 2D flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} = 0$$

- Substituting the clever transformation

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x}$$

- Gives

$$\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial y \partial x} \equiv 0$$

This is true for any smooth function $\psi(x,y)$

Potential function (ϕ)

- If the curl of a vector is zero, the vector can be expressed as the gradient of a scalar function ϕ, called the potential function.

Vector identity: $\vec{\nabla} \times \vec{\nabla}\phi = 0$ **thus if** $\vec{\nabla} \times \vec{V} = 0$, **then** $\vec{V} = \vec{\nabla}\phi$

In fluid mechanics, vector $\vec{V}$ is the velocity vector, the curl of which is the vorticity vector ζ and thus we call ϕ the velocity potential.

Mathematically $\phi = f(x,y,z,t)$ ---- unsteady flow

$\phi = f(x,y,z)$ ---- steady flow

$$u = \frac{\partial \phi}{\partial x} \; ; \; v = \frac{\partial \phi}{\partial y} \; ; \; w = \frac{\partial \phi}{\partial z}$$

(OR)

$$u = -\frac{\partial \phi}{\partial x} \; ; \; v = -\frac{\partial \phi}{\partial y} \; ; \; w = -\frac{\partial \phi}{\partial z}$$

- For an incompressible steady flow the continuity equation is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

- Substituting the value of u,v,w in terms of ϕ in above equation, we obtain the Laplace equation

$$\frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial y}\right)+\frac{\partial}{\partial z}\left(-\frac{\partial \phi}{\partial z}\right)=0$$

$$\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial y^2}+\frac{\partial^2 \phi}{\partial z^2}=0$$

- If the velocity potential satisfies the Laplace equation it represents the possible steady, incompressible, irrotational flow. Often an irrotational flow is known as potential flow

Important Remarks about ψ and ϕ

" The stream function is defined by continuity; the Laplace equations for ψ results from irrotationality"

"The velocity potential is defined by irrotationality; the Laplace equations for ϕ results from continuity"

Streamline & Equipotential Line relationship

- Curves of constant values of ψ define streamlines of the flow .
- Curves of constant values of ϕ define equipotential lines of the flow.
- In Planar irrotational flow the streamlines and equipotential lines are intersect each other at right angles.
- Solutions of ψ and ϕ are called harmonic functions.

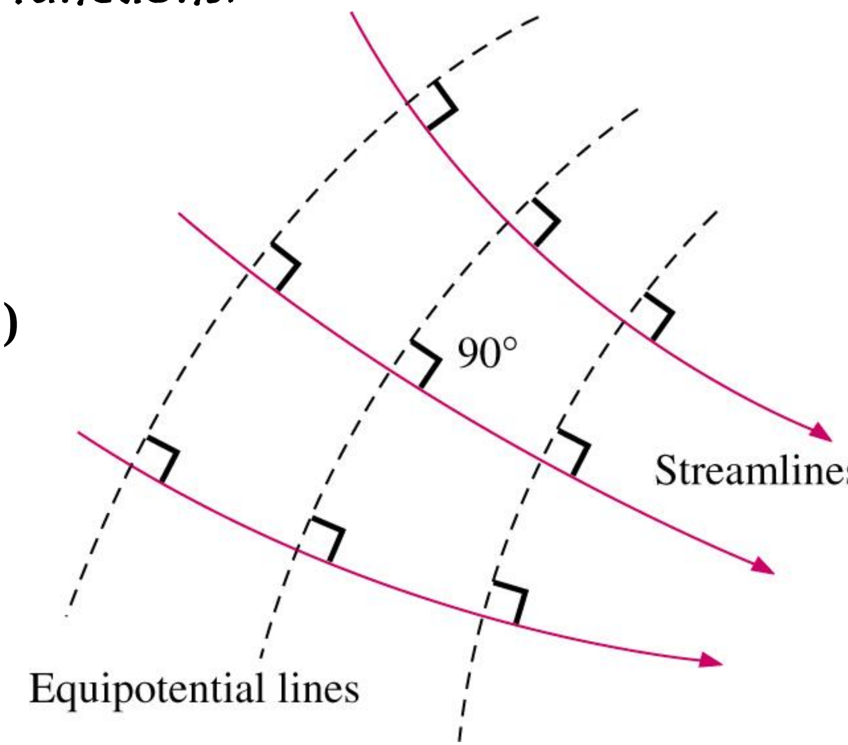
$$\textbf{For equipotential line}, \phi = \text{constant}, \; d\phi = 0$$

$$\textbf{but } \phi = f(x, y) \textbf{ for steady flow}$$

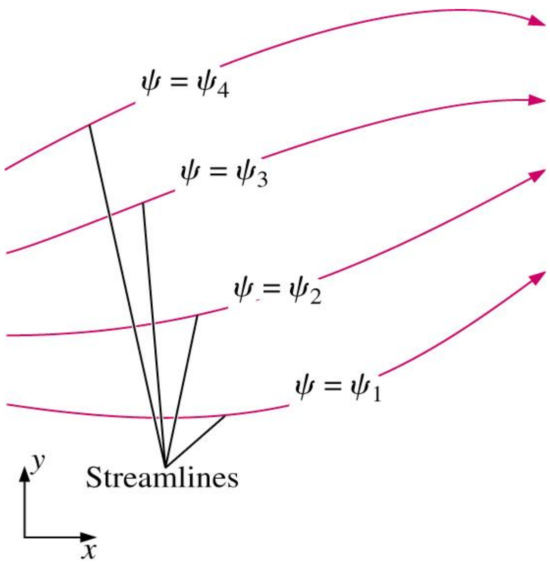
$$d\phi = \frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy = -udx - vdy = -(udx + vdy)$$

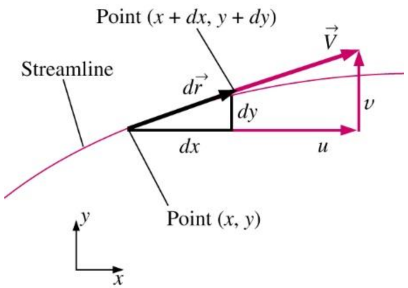
$$\textbf{for equipotential line } d\phi = 0 = -(udx + vdy)$$

$$\textbf{slope of the equipotential line } \frac{dy}{dx} = -\frac{u}{v}$$



The Stream Function
Physical Significance





- Recall

along a streamline

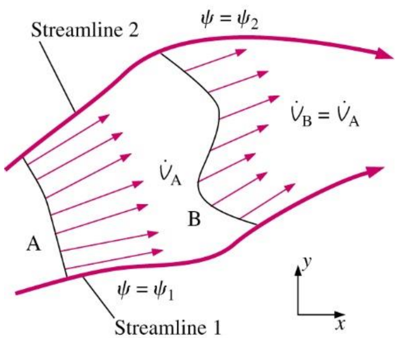
$$\frac{dy}{dx} = \frac{v}{u} \Rightarrow -v\,dx + u\,dy = 0$$

$$\frac{\partial \psi}{\partial x}\,dx + \frac{\partial \psi}{\partial y}\,dy = 0$$

$$d\psi = 0$$

∴ Change in ψ along streamline is zero

The Stream Function
Physical Significance



- Difference in ψ between streamlines is equal to volume flow rate between streamlines

$$\dot{V}_A = \dot{V}_B = \psi_2 - \psi_1$$

Cauchy Riemann Equation

- From the above discussions the following conclusions are arrived:
- Potential function exist only for irrotational flow
- Stream function applies to both rotational and irrotational flows
- In irrotational flow both ψ & ϕ satisfy the Laplace equation as they are interchangeable.

$$\mathbf{u} = -\frac{\partial \phi}{\partial \mathbf{x}} = \frac{\partial \Psi}{\partial \mathbf{y}}$$

CR Equation

$$\mathbf{v} = -\frac{\partial \phi}{\partial \mathbf{y}} = -\frac{\partial \Psi}{\partial \mathbf{x}}$$

Relation between Stream Function and Velocity Potential

- Let two curves $\psi = C$ & $\phi = C$ intersect each other at any point . At the point of intersection the slopes are :

For $\phi = C$: slope = $$\frac{\partial y}{\partial x} = \frac{\frac{\partial \Psi}{\partial x}}{\frac{\partial \Psi}{\partial y}} = \frac{-v}{u} = -\frac{v}{u}$$

For $\psi = C$: slope = $$\frac{\partial y}{\partial x} = \frac{\frac{\partial \phi}{\partial x}}{\frac{\partial \phi}{\partial y}} = \frac{-u}{-v} = \frac{u}{v}$$

$$\frac{\partial y}{\partial x} = \frac{u}{v} x - \frac{v}{u} = -1$$

- It shows that these two sets of curves intersect each other orthogonally at points of intersection.

RTT, Mass, Bernoulli, and Energy Equations

Chapter 5

Introduction

Reynolds Transport Theorem (RTT) provides a link between the system approach and the control volume approach

Three equations which are commonly used in fluid mechanics

- The mass equation is an expression of the conservation of mass principle.
- The Bernoulli equation is concerned with the conservation of kinetic, potential, and flow energies of a fluid stream and their conversion to each other.
- The energy equation is a statement of the conservation of energy principle.

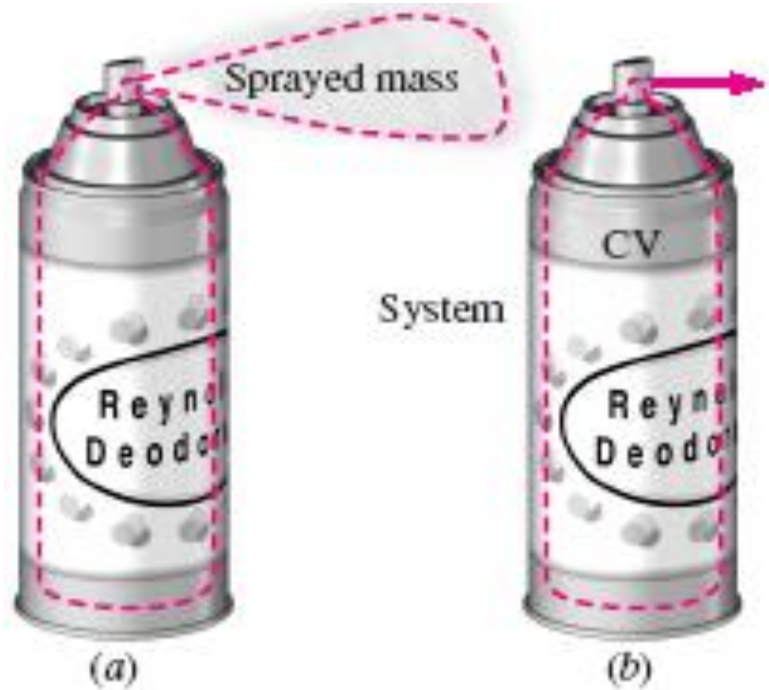
Objectives

<u>After completing this chapter, you should be able to</u>

- Understand the usefulness of the Reynolds Transport Theorem
- Apply the mass equation to balance the incoming and outgoing flow rates in a flow system.
- Recognize various forms of mechanical energy, and work with energy conversion efficiencies.
- Understand the use and limitations of the Bernoulli equation, and apply it to solve a variety of fluid flow problems.
- Work with the energy equation expressed in terms of heads, and use it to determine turbine power output and pumping power requirements.

Reynolds Transport Theorem (RTT)

A system is a quantity of matter of fixed identity. No mass can cross a system boundary.



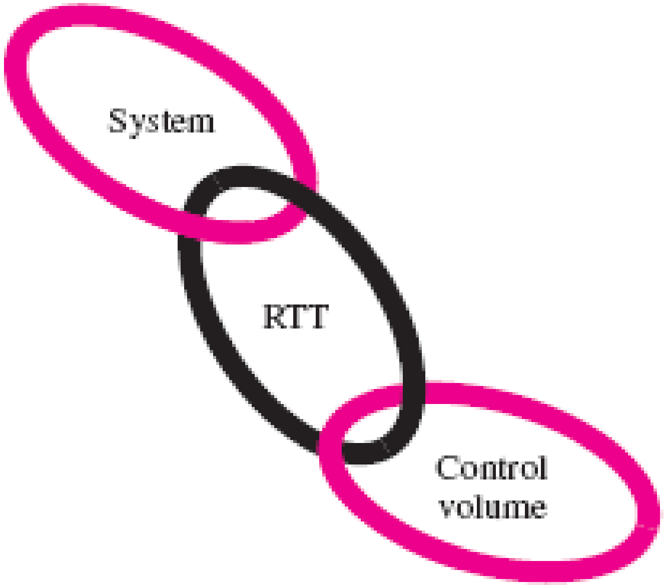
A control volume is a region in space chosen for study. Mass can cross a control surface.

System deformable

CV fixed, nondeformable

Reynolds Transport Theorem (RTT)

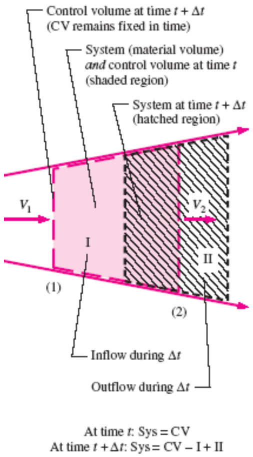
The fundamental conservation laws (conservation of mass, energy, and momentum) apply directly to systems.



However, in most fluid mechanics problems, control volume analysis is preferred over system analysis (for the same reason that the Eulerian description is usually preferred over the Lagrangian description).

Therefore, we need to transform the conservation laws from a system to a control volume. This is accomplished with the Reynolds transport theorem (RTT).

Reynolds Transport Theorem (RTT)



$$B_{sys,\,t} = B_{CV,\,t} \qquad \text{(the system and CV concide at time } t\text{)}$$

$$B_{sys,\,t+\Delta t} = B_{CV,\,t+\Delta t} - B_{I,\,t+\Delta t} + B_{II,\,t+\Delta t}$$

$$\frac{B_{sys,\,t+\Delta t} - B_{sys,\,t}}{\Delta t} = \frac{B_{CV,\,t+\Delta t} - B_{CV,\,t}}{\Delta t} - \frac{B_{I,\,t+\Delta t}}{\Delta t} + \frac{B_{II,\,t+\Delta t}}{\Delta t}$$

$$\frac{dB_{sys}}{dt} = \frac{dB_{CV}}{dt} - \dot{B}_{in} + \dot{B}_{out}$$

$$B_{I,\,t+\Delta t} = b_1 m_{I,\,t+\Delta t} = b_1 \rho_1 V_{I,\,t+\Delta t} = b_1 \rho_1 V_1 \,\Delta t\, A_1$$

$$B_{II,\,t+\Delta t} = b_2 m_{II,\,t+\Delta t} = b_2 \rho_2 V_{II,\,t+\Delta t} = b_2 \rho_2 V_2 \,\Delta t\, A_2$$

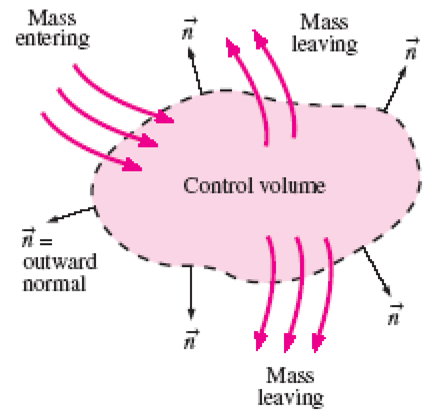
$$\dot{B}_{in} = \dot{B}_{I} = \lim_{\Delta t \to 0} \frac{B_{I,\,t+\Delta t}}{\Delta t} = \lim_{\Delta t \to 0} \frac{b_1 \rho_1 V_1 \,\Delta t\, A_1}{\Delta t} = b_1 \rho_1 V_1 A_1$$

$$\dot{B}_{out} = \dot{B}_{II} = \lim_{\Delta t \to 0} \frac{B_{II,\,t+\Delta t}}{\Delta t} = \lim_{\Delta t \to 0} \frac{b_2 \rho_2 V_2 \,\Delta t\, A_2}{\Delta t} = b_2 \rho_2 V_2 A_2$$

$$\frac{dB_{sys}}{dt} = \frac{dB_{CV}}{dt} - b_1 \rho_1 V_1 A_1 + b_2 \rho_2 V_2 A_2$$

Reynolds Transport Theorem (RTT)

the time rate of change of the property B of the system is equal to the time rate of change of B of the control volume plus the net flux of B out of the control volume by mass crossing the control surface.



$$\dot{B}_{\text{net}} = \dot{B}_{\text{out}} - \dot{B}_{\text{in}} = \int_{\text{CS}} \rho b \vec{V} \cdot \vec{n} \, dA \quad \text{(inflow if negative)}$$

Reynolds Transport Theorem (RTT)

The total amount of property B within the control volume must be determined by integration:

$$B_{\text{CV}} = \int_{\text{CV}} \rho b \, dV$$

Therefore, the system-to-control- volume transformation for a fixed control volume:

$$\frac{dB_{\text{sys}}}{dt} = \frac{d}{dt}\int_{\text{CV}} \rho b \, dV + \int_{\text{CS}} \rho b \vec{V} \cdot \vec{n} \, dA$$

Reynolds Transport Theorem (RTT)

Material derivative (differential analysis): $\frac{Db}{Dt} = \frac{\partial b}{\partial t} + \left(\vec{V} \bullet \vec{\nabla}\right) b$

General RTT, nonfixed CV (integral analysis):

$$\frac{dB_{\text{sys}}}{dt} = \frac{d}{dt} \int_{\text{CV}} \rho b \, dV + \int_{\text{CS}} \rho b \vec{V}_r \cdot \vec{n} \, dA$$

	Mass	Momentum	Energy	Angular momentum
B, Extensive properties	m	$m\vec{V}$	E	$\vec{H}$
b, Intensive properties	1	$\vec{V}$	e	$\left(\vec{r} \times \vec{V}\right)$

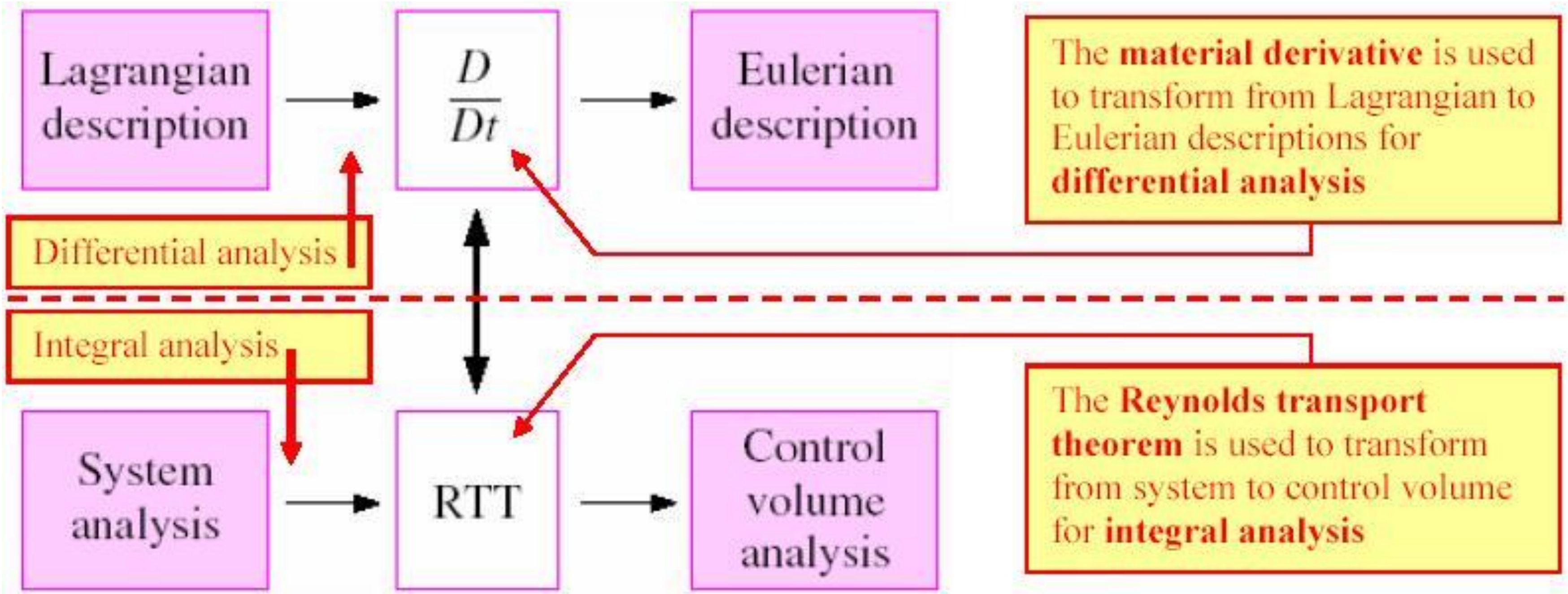
we can apply RTT to conservation of mass, energy, linear momentum, and angular momentum.

Reynolds Transport Theorem (RTT)

- Interpretation of the RTT:
  - Time rate of change of the property B of the system is equal to (Term 1) + (Term 2)
  - Term 1: the time rate of change of B of the control volume
  - Term 2: the net flux of B out of the control volume by mass crossing the control surface

$$\frac{dB_{sys}}{dt} = \int_{CV} \frac{\partial}{\partial t}(\rho b)\, dV + \int_{CS} \rho b \vec{V} \cdot \vec{n}\, dA$$

Reynolds Transport Theorem (RTT)



There is a direct analogy between the transformation from Lagrangian to Eulerian descriptions (for differential analysis using infinitesimally small fluid elements) and the transformation from systems to control volumes (for integral analysis using large, finite flow fields).

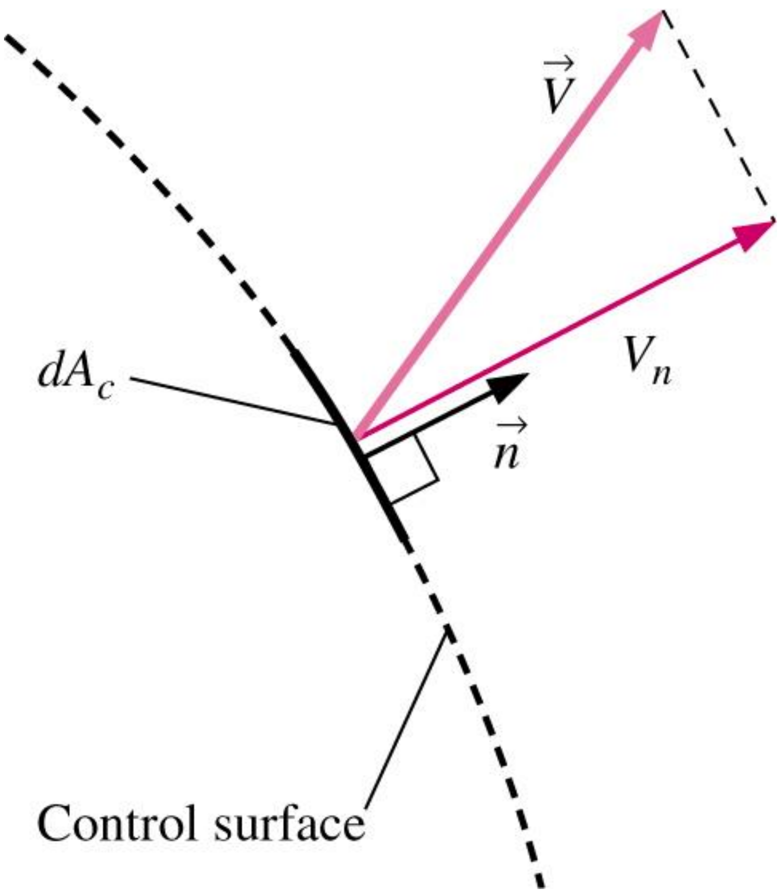
Remarks about RTT

- The RTT is useful for transforming conservation equations from their naturally occurring systems forms to their control volume.
- The RTT can be applied to any control volume, fixed, moving, or deforming.
- The RTT has an unsteady term and can be applied to unsteady problems.
- The extensive property B (or its intensive form b) in the RTT can be any property of the fluid – scalar, vector, or even tensor.

Conservation of Mass

- Conservation of mass principle is one of the most fundamental principles in nature.
- Mass, like energy, is a conserved property, and it cannot be created or destroyed during a process.
- For closed systems mass conservation is implicit since the mass of the system remains constant during a process.
- For control volumes, mass can cross the boundaries which means that we must keep track of the amount of mass entering and leaving the control volume.

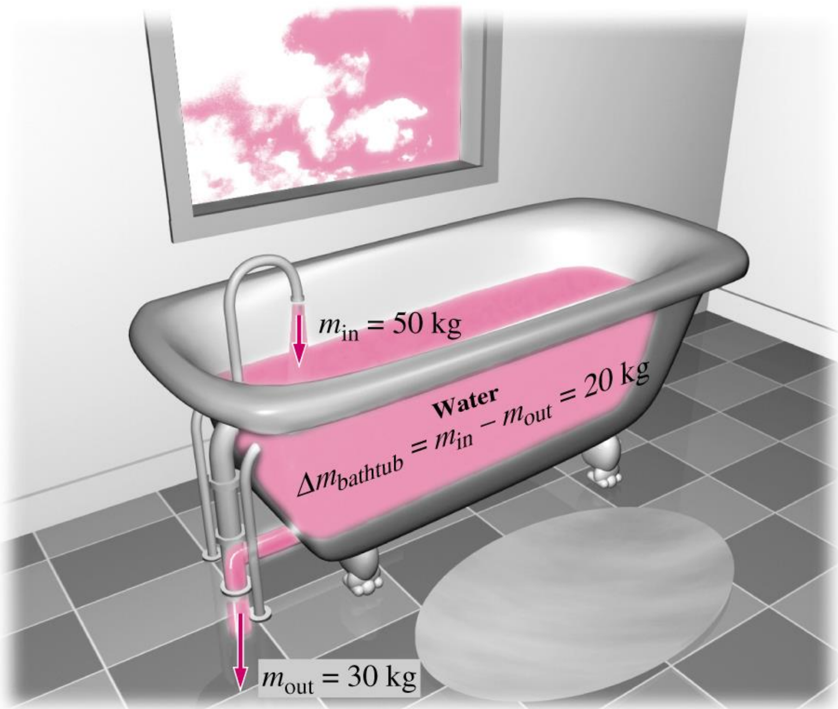
Mass Flow Rate



$$\dot{m} = \int_{A_c} \delta \dot{m} = \int_{A_c} \rho V_n dA_c$$

- The amount of mass flowing through a control surface per unit time is called the mass flow rate and is denoted $\dot{m}$
- The dot over a symbol is used to indicate time rate of change.
- Flow rate across the entire cross-sectional area of a pipe or duct is obtained by integration

Conservation of Mass Principle

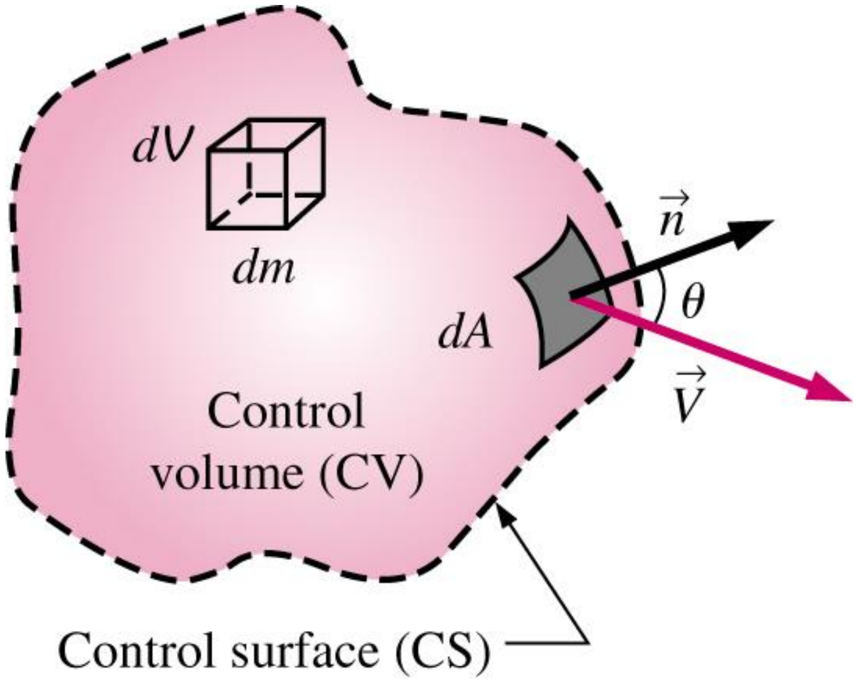


$m_{in} - m_{out} = \Delta m_{CV}$ (kg)

- The conservation of mass principle can be expressed as

$$\dot{m}_{in} - \dot{m}_{out} = \frac{dm_{CV}}{dt}$$

- Where $\dot{m}_{in}$ and $\dot{m}_{out}$ are the total rates of mass flow into and out of the CV, and dm_{CV}/dt is the rate of change of mass within the CV.



$$\vec{V} \bullet \vec{n} = |\vec{V}||\vec{n}| \cos\theta$$

$$= V \cos\theta$$

$$\text{if } \theta < 90, \cos\theta > 0 \Rightarrow \text{outflow}$$

$$\text{if } \theta > 90, \cos\theta < 0 \Rightarrow \text{inflow}$$

$$\text{if } \theta = 90 \cos\theta = 0 \Rightarrow \text{no flow}$$

$$\vec{V} \bullet \vec{n} = |\vec{V}||\vec{n}| \cos\theta$$

$$= V \cos\theta$$

$$\text{if } \theta = 0 \Rightarrow \cos\theta = 1 \Rightarrow \text{maximum outflow}$$

$$\text{if } \theta = 180 \Rightarrow \cos\theta = -1 \Rightarrow \text{minimum inflow}$$

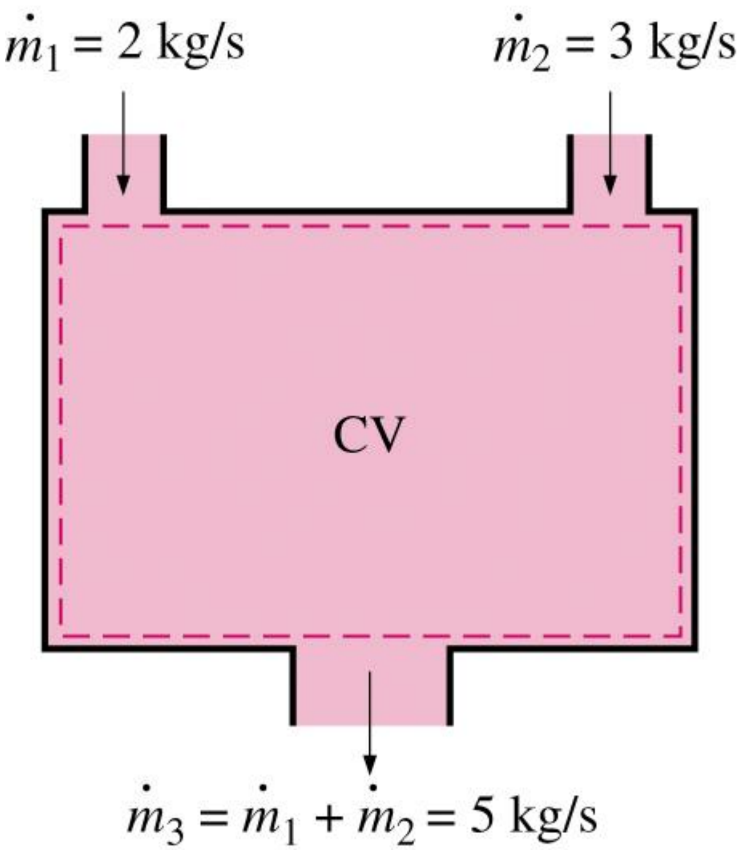
$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho b \, dV + \int_{CS} \rho b(\vec{V} \cdot \vec{n})\, dA$$

$B = m$ $\qquad$ $b = 1$ $\qquad$ $b = 1$

$$\frac{dm_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho \, dV + \int_{CS} \rho(\vec{V} \cdot \vec{n})\, dA$$

The conservation of mass equation is obtained by replacing B in the Reynolds transport theorem by mass m, and b by 1 (m per unit mass $= m/m = 1$).

Steady-Flow Processes



- For steady flow, the total amount of mass contained in CV is constant.
- Total amount of mass entering must be equal to total amount of mass leaving

$$\sum_{in} \dot{m} = \sum_{out} \dot{m}$$

- For incompressible flows,

$$\sum_{in} V_n A_n = \sum_{out} V_n A_n$$

Mechanical Energy

- Mechanical energy can be defined as the form of energy that can be converted to mechanical work completely by an ideal mechanical device.

- Flow P/ρ, kinetic V^2/g, and potential gz energy are the forms of mechanical energy $e_{mech} = P/\rho + V^2/g + gz$

- Mechanical energy change of a fluid during incompressible flow becomes

$$\Delta e_{mech} = \frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g\left(z_2 - z_1\right)$$

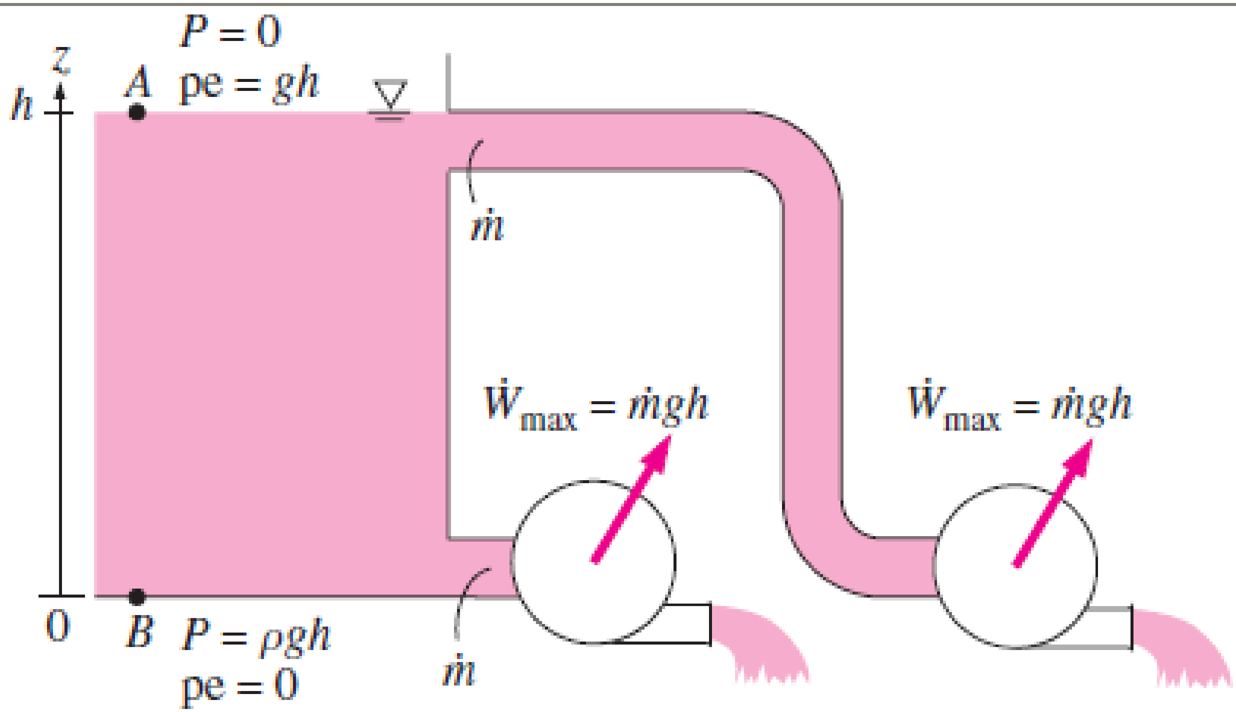
- In the absence of loses, Δe_{mech} represents the work supplied to the fluid ($\Delta e_{mech} > 0$) or extracted from the fluid ($\Delta e_{mech} < 0$).

Efficiency

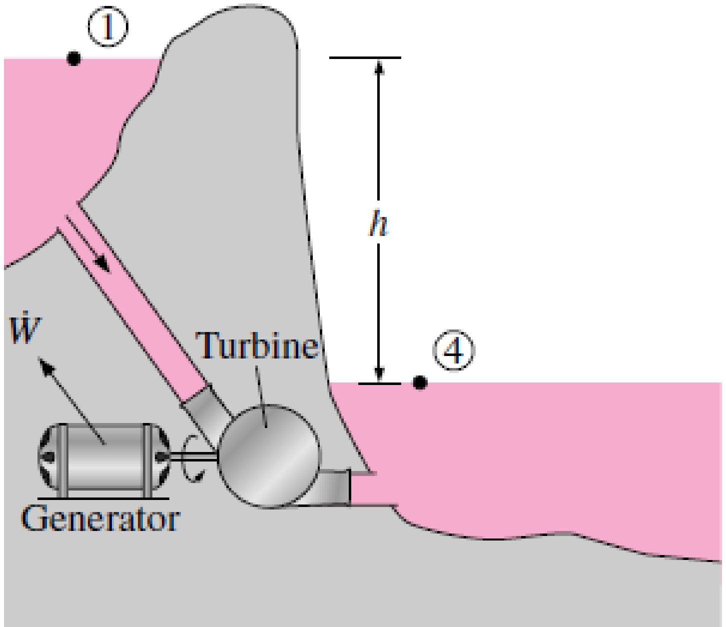
- Transfer of e_{mech} is usually accomplished by a rotating shaft: shaft work
- Pump, fan, propulsion: receives shaft work (e.g., from an electric motor) and transfers it to the fluid as mechanical energy
- Turbine: converts e_{mech} of a fluid to shaft work.
- In the absence of irreversibilities (e.g., friction), mechanical efficiency of a device or process can be defined as

$$\eta_{mech} = \frac{E_{mech,out}}{E_{mech,in}} = 1 - \frac{E_{mech,loss}}{E_{mech,in}}$$

- If $\eta_{mech} < 100\%$, losses have occurred during conversion.

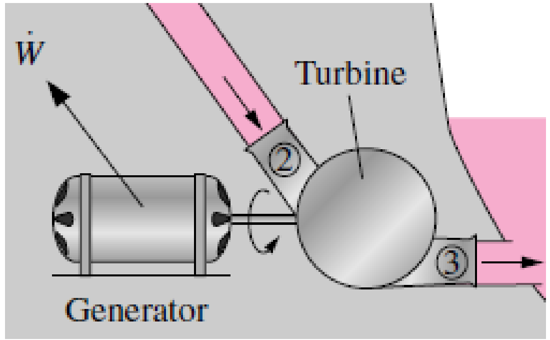


The mechanical energy of water at the bottom of a container is equal to the mechanical energy at any depth including the free surface of the container.



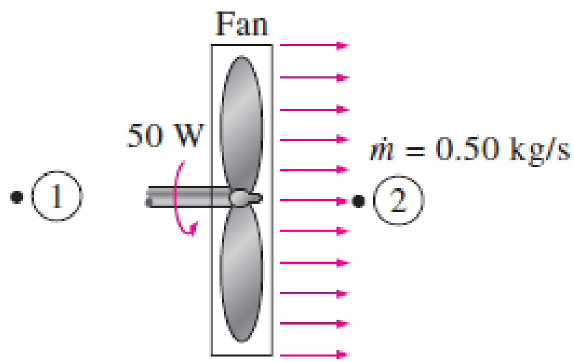
$$\dot{W}_{max} = \dot{m}\Delta e_{mech} = \dot{m}g(z_1 - z_4) = \dot{m}gh$$

since $P_1 \approx P_4 = P_{atm}$ and $V_1 = V_4 \approx 0$



$$\dot{W}_{max} = \dot{m}\Delta e_{mech} = \dot{m}\frac{(P_2 - P_3)}{\rho} = \dot{m}\frac{\Delta P}{\rho}$$

since $V_2 \approx V_3$ and $z_2 \approx z_3$



$V_1 = 0$, $V_2 = 12$ m/s

$z_1 = z_2$

$P_1 = P_2$

$$\eta_{mech,\,fan} = \frac{\Delta \dot{E}_{mech,\,fluid}}{\dot{W}_{shaft,\,in}} = \frac{\dot{m}V_2^2/2}{\dot{W}_{shaft,\,in}}$$

$$= \frac{(0.50\ \text{kg/s})(12\ \text{m/s})^2/2}{50\ \text{W}}$$

$$= 0.72$$

Motor: $$\eta_{\text{motor}} = \frac{\text{Mechanical power output}}{\text{Electric power input}} = \frac{W_{\text{shaft, out}}}{\dot{W}_{\text{elect, in}}}$$

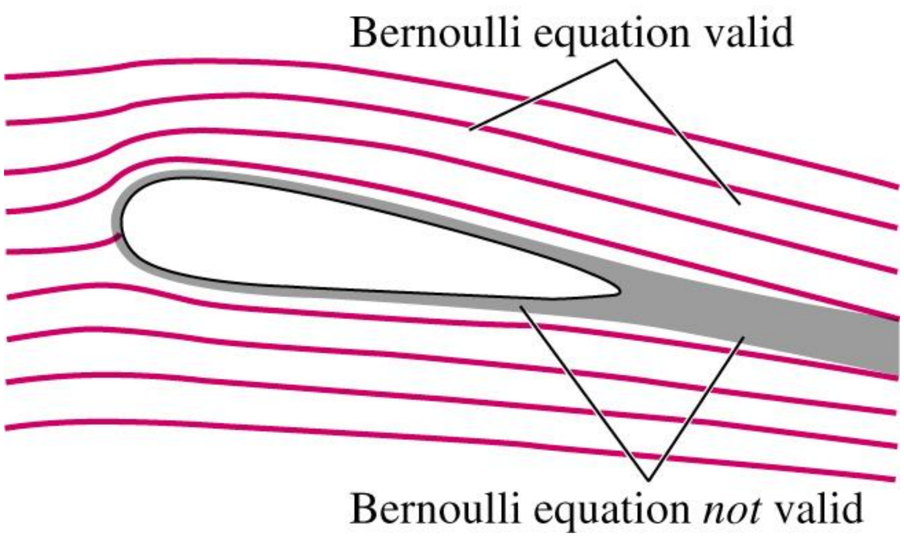
and

Generator: $$\eta_{\text{generator}} = \frac{\text{Electric power output}}{\text{Mechanical power input}} = \frac{\dot{W}_{\text{elect, out}}}{\dot{W}_{\text{shaft, in}}}$$

$$\eta_{\text{pump–motor}} = \eta_{\text{pump}}\,\eta_{\text{motor}} = \frac{\dot{W}_{\text{pump, }u}}{\dot{W}_{\text{elect, in}}} = \frac{\Delta \dot{E}_{\text{mech, fluid}}}{\dot{W}_{\text{elect, in}}}$$

$$\eta_{\text{turbine–gen}} = \eta_{\text{turbine}}\,\eta_{\text{generator}} = \frac{\dot{W}_{\text{elect, out}}}{\dot{W}_{\text{turbine, }e}} = \frac{\dot{W}_{\text{elect, out}}}{|\Delta \dot{E}_{\text{mech, fluid}}|}$$

The Bernoulli Equation



- The Bernoulli equation is an approximate relation between pressure, velocity, and elevation and is valid in regions of steady, incompressible flow where net frictional forces are negligible.

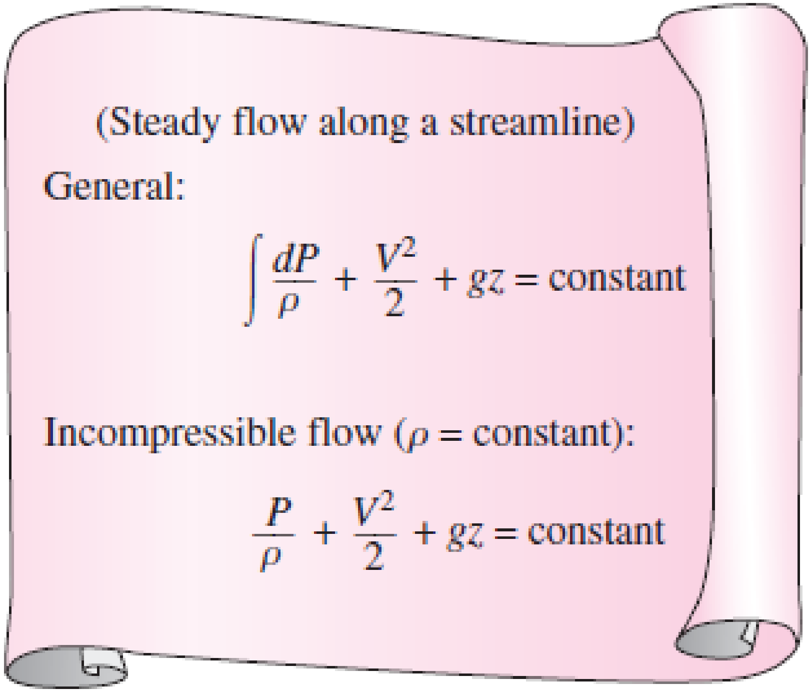
- Equation is useful in flow regions outside of boundary layers and wakes.

The Bernoulli Equation

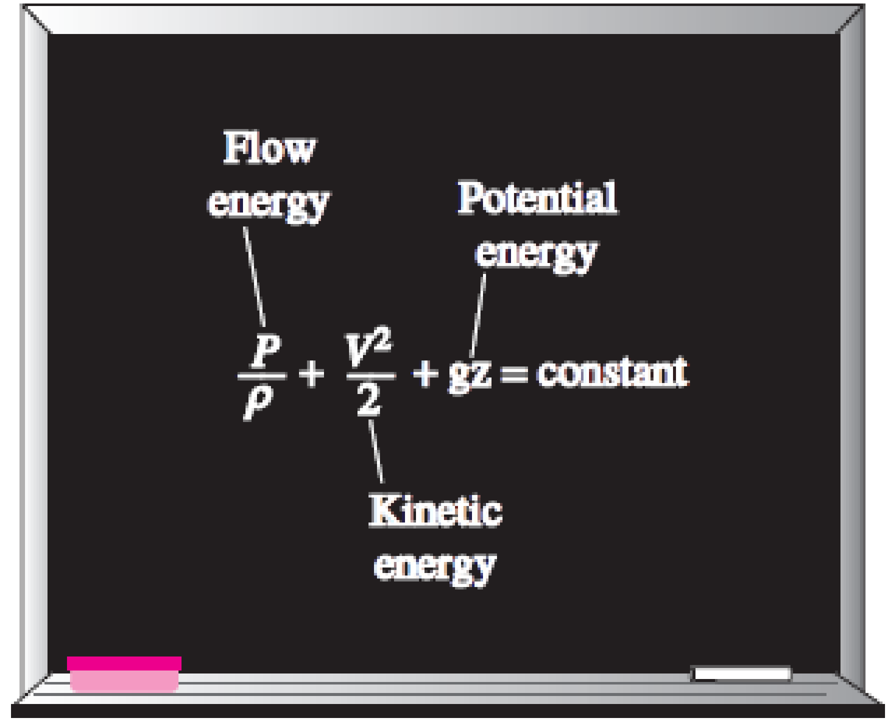
- If we neglect piping losses, and have a system without pumps or turbines

$$\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2$$

- This is the Bernoulli equation
- 3 terms correspond to : Static, dynamic, and hydrostatic head (or pressure).

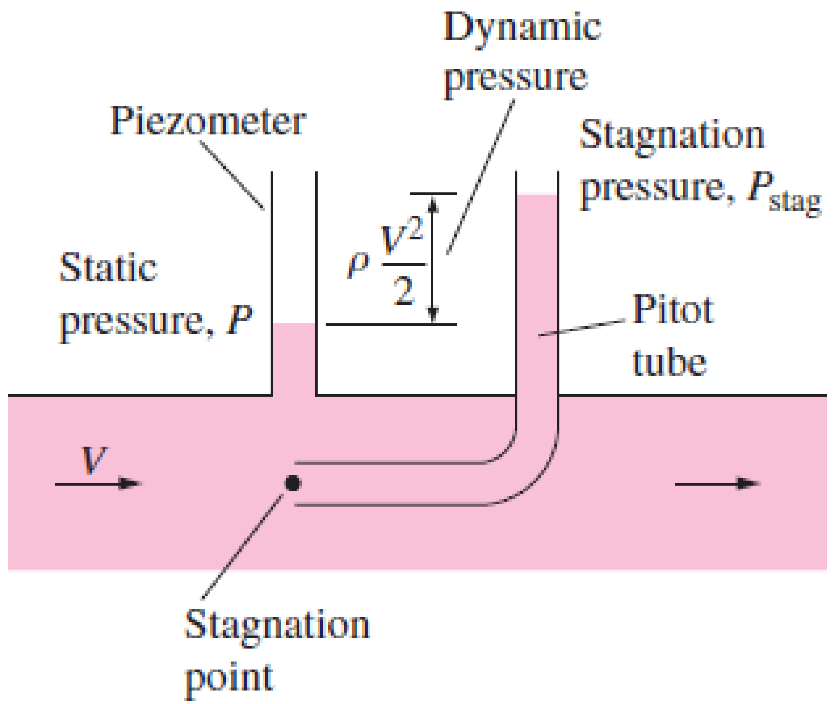


The Bernoulli equation is derived assuming incompressible flow, and thus it should not be used for flows with significant compressibility effects.



The Bernoulli equation states that the sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow.

Static, Dynamic and Stagnation Pressures



$$V = \sqrt{\frac{2(P_{stag} - P)}{\rho}}$$

The static, dynamic, and stagnation pressures.

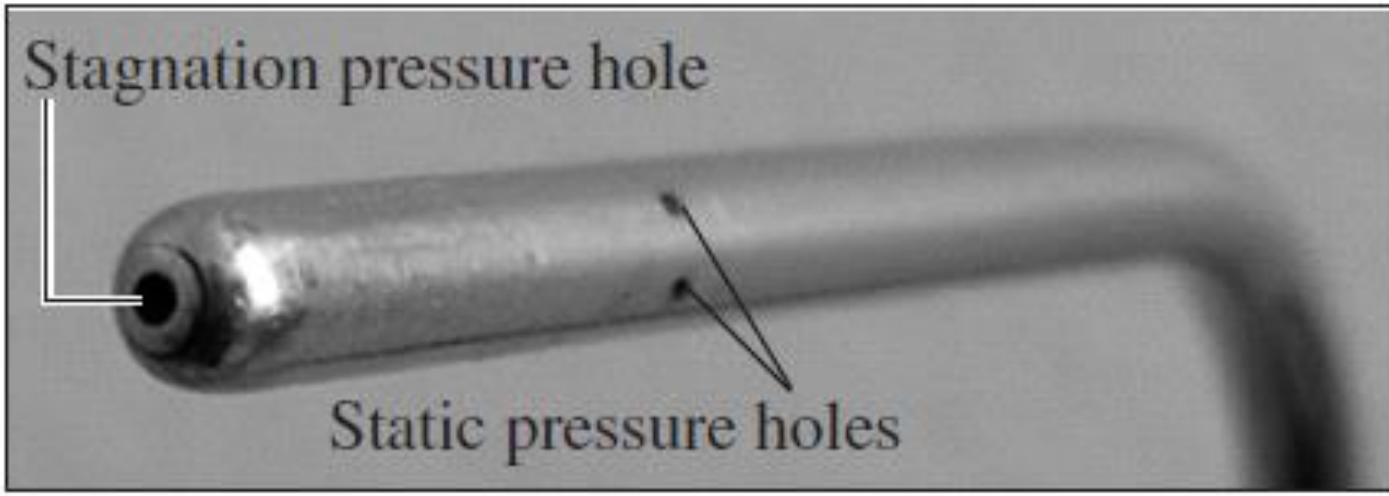
$$P + \rho \frac{V^2}{2} + \rho g z = \text{constant} \qquad \text{(kPa)}$$

P is the static pressure (it does not incorporate any dynamic effects); it represents the actual pressure of the fluid. This is the same as the pressure used in thermodynamics and property tables.

$\rho V^2/2$ is the dynamic pressure; it represents the pressure rise when the fluid in motion is brought to a stop isentropically.

$\rho g z$ is the hydrostatic pressure, which is not pressure in a real sense since its value depends on the reference level selected; it accounts for the elevation effects, i.e., of fluid weight on pressure.

The sum of the static, dynamic, and hydrostatic pressures is called the total pressure. Therefore, the Bernoulli equation states that the total pressure along a streamline is constant.

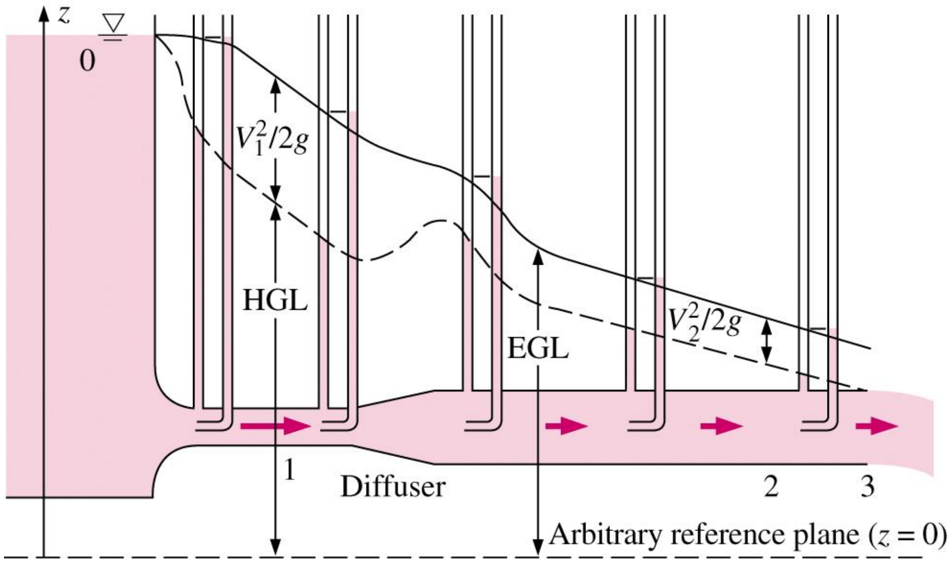


Close-up of a Pitot-static probe, showing the stagnation pressure hole and two of the five static circumferential pressure holes.

The Bernoulli Equation

- Limitations on the use of the Bernoulli Equation
  - Steady flow: d/dt = 0
  - Frictionless flow
  - No shaft work: $w_{pump}=w_{turbine}= 0$
  - Incompressible flow: ρ = constant
  - No heat transfer: $q_{net,in}=0$
  - Applied along a streamline

HGL and EGL



- It is often convenient to plot mechanical energy graphically using heights to facilitate visualization of the various terms of the Bernoulli equation.

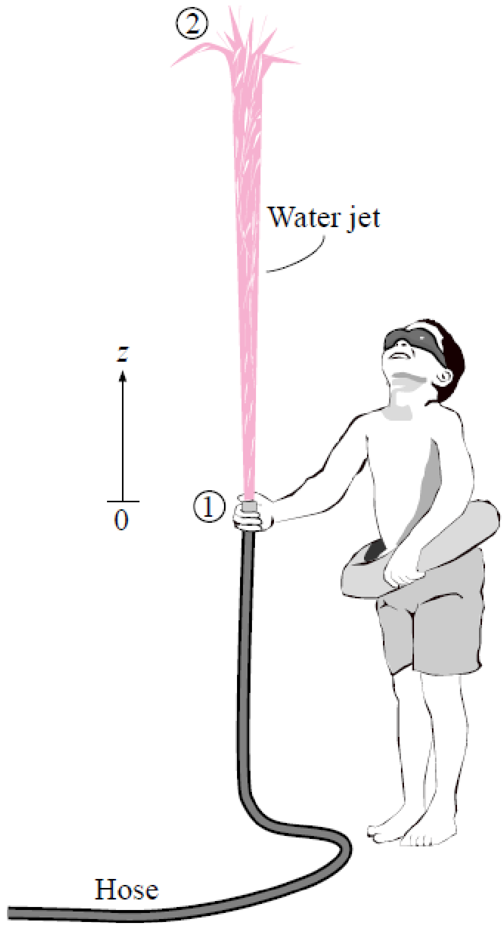
- Hydraulic Grade Line

$$HGL = \frac{P}{\rho g} + z$$

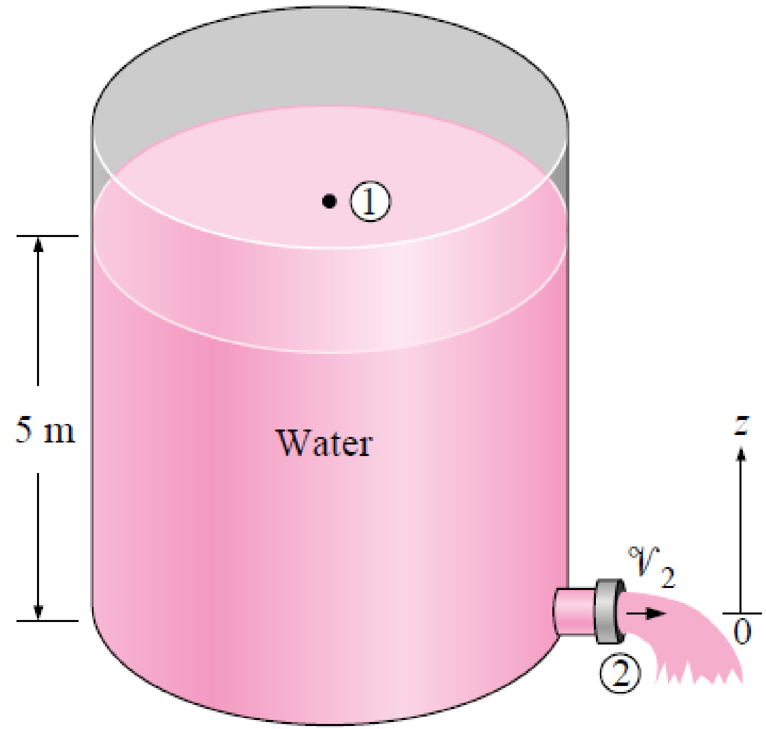
- Energy Grade Line (or total energy)

$$EGL = \frac{P}{\rho g} + \frac{V^2}{2g} + z$$

Applications Of The Bernoulli Equation

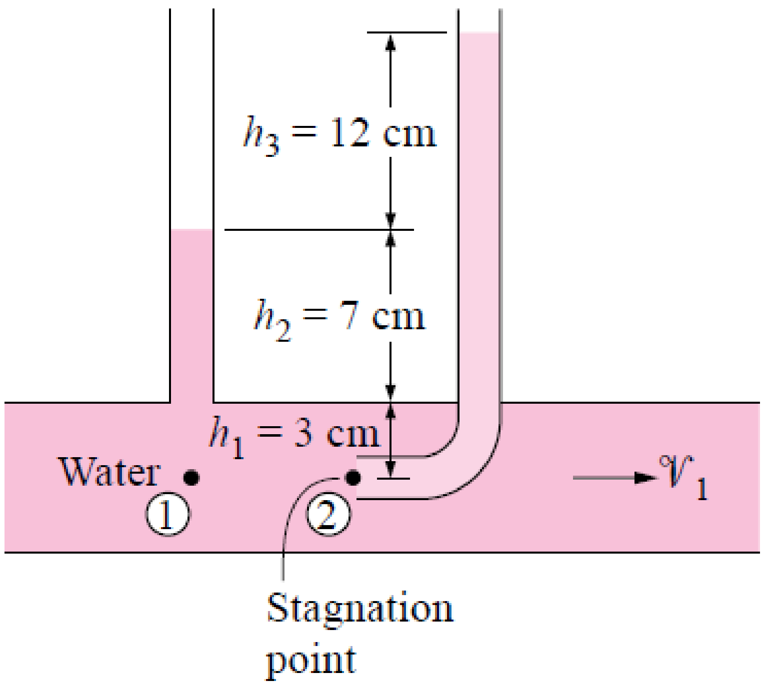


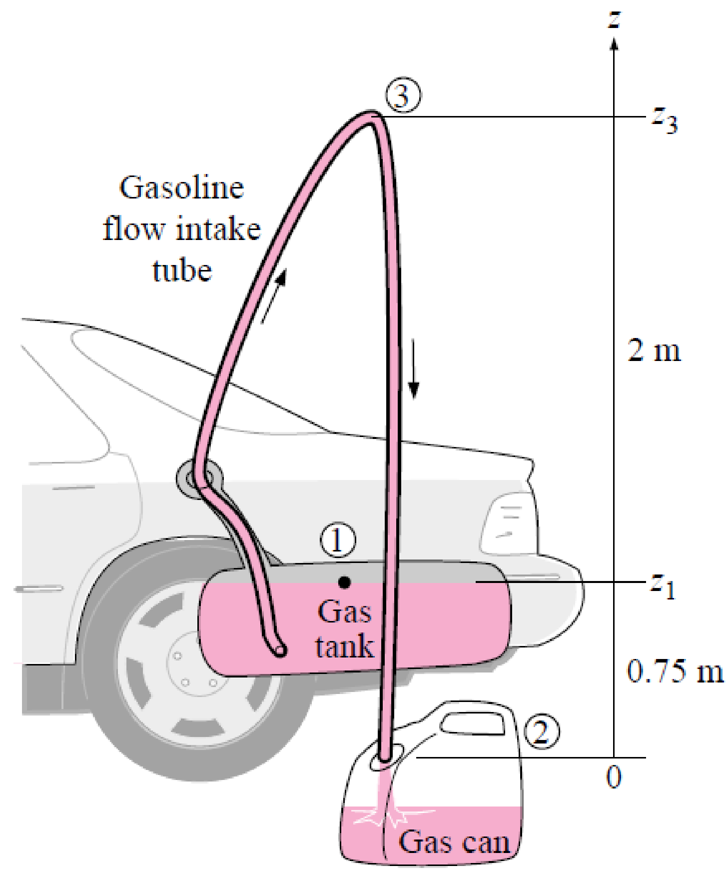
Spraying Water into the Air



Water Discharge from a Large Tank

Velocity Measurement by a Pitot Tube

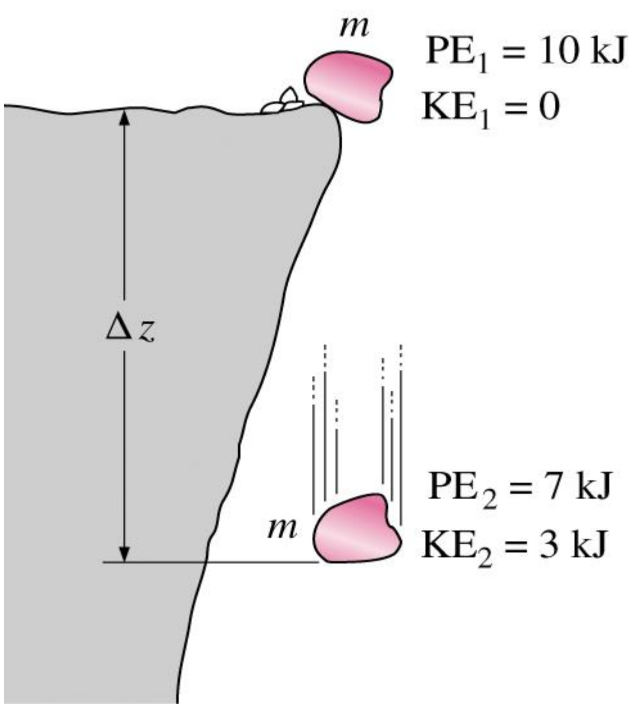




Siphoning out Gasoline from a Fuel Tank

General Energy Equation

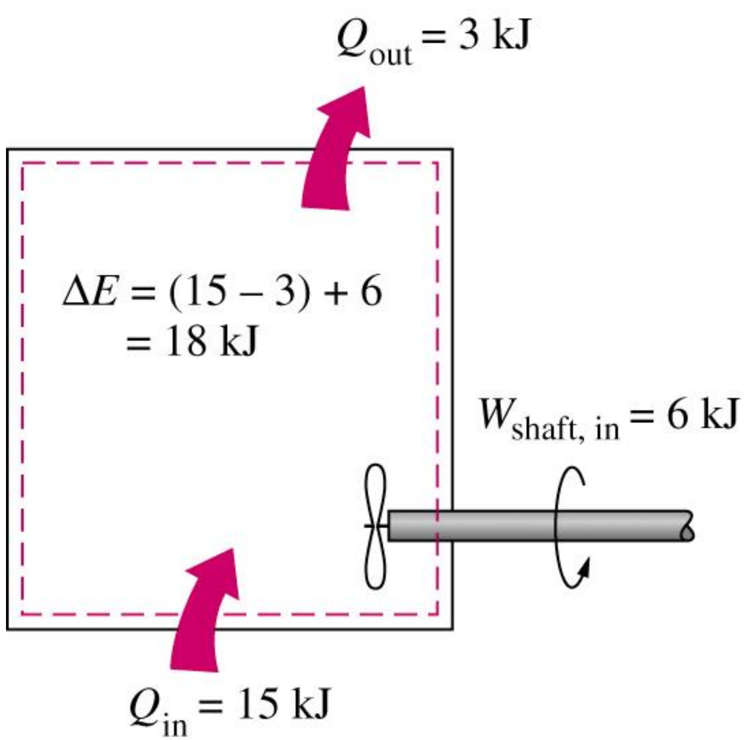
- One of the most fundamental laws in nature is the 1st law of thermodynamics, which is also known as the conservation of energy principle.
- It states that energy can be neither created nor destroyed during a process; it can only change forms



- Falling rock, picks up speed as PE is converted to KE.
- If air resistance is neglected,

$$PE + KE = constant$$

General Energy Equation



- The energy content of a closed system can be changed by two mechanisms: heat transfer Q and work transfer W.

- Conservation of energy for a closed system can be expressed in rate form as

$$\dot{Q}_{net,in} + \dot{W}_{net,in} = \frac{dE_{sys}}{dt}$$

- Net rate of heat transfer to the system:

$$\dot{Q}_{net,in} = \dot{Q}_{in} - \dot{Q}_{out}$$

- Net power input to the system:

$$\dot{W}_{net,in} = \dot{W}_{in} - \dot{W}_{out}$$

General Energy Equation

- Recall general RTT

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho b \, dV + \int_{CS} \rho b \left(\vec{V}_r \bullet \vec{n}\right) dA$$

- "Derive" energy equation using B=E and b=e

$$\frac{dE_{sys}}{dt} = \dot{Q}_{net,in} + \dot{W}_{net,in} = \frac{d}{dt}\int_{CV} \rho e \, dV + \int_{CS} \rho e \left(\vec{V}_r \bullet \vec{n}\right) dA$$

- Break power into rate of shaft and pressure work

$$\dot{W}_{net,in} = \dot{W}_{shaft,net,in} + \dot{W}_{pressure,net,in} = \dot{W}_{shaft,net,in} - \int P \left(\vec{V} \bullet \vec{n}\right) dA$$

General Energy Equation

- Moving integral for rate of pressure work to RHS of energy equation results in:

$$\dot{Q}_{net,in} + \dot{W}_{shaft,net,in} = \frac{d}{dt}\int_{CV} \rho e \, d\forall + \int_{CS} \left(\frac{P}{\rho} + e \right) e \left(\vec{V}_r \cdot \vec{n} \right) dA$$

- Recall that P/ρ is the flow work, which is the work associated with pushing a fluid into or out of a CV per unit mass.

General Energy Equation

- As with the mass equation, practical analysis is often facilitated as averages across inlets and exits

$$\dot{Q}_{net,in} + \dot{W}_{shaft,net,in} = \frac{d}{dt}\int_{CV} \rho e \, d\mathcal{V} + \sum_{out} \dot{m}\left(\frac{P}{\rho} + e\right) - \sum_{in} \dot{m}\left(\frac{P}{\rho} + e\right)$$

- Since e=u+ke+pe = u+V^2/2+gz

$$\dot{Q}_{net,in} + \dot{W}_{shaft,net,in} = \frac{d}{dt}\int_{CV} \rho e \, d\mathcal{V} + \sum_{out} \dot{m}\left(\frac{P}{\rho} + u + \frac{V^2}{2} + gz\right) - \sum_{in} \dot{m}\left(\frac{P}{\rho} + u + \frac{V^2}{2} + gz\right)$$

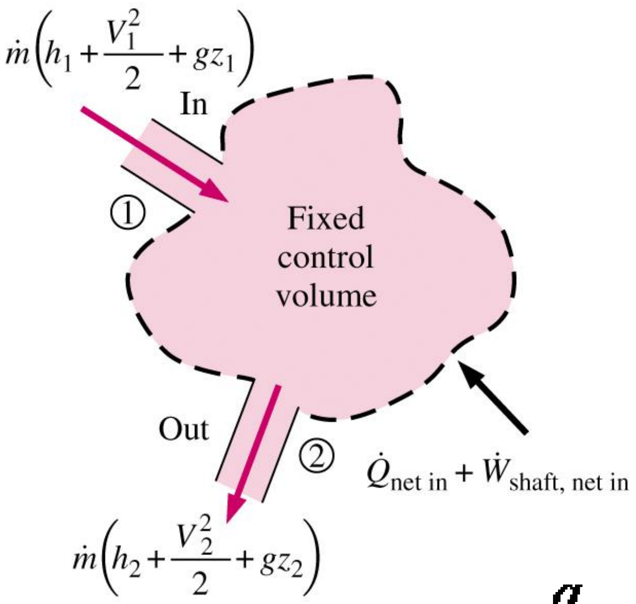
Energy Analysis of Steady Flows

$$Q_{net,in} + W_{shaft,net,in} = \sum_{out} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{in} \dot{m}\left(h + \frac{V^2}{2} + gz\right)$$

- For steady flow, time rate of change of the energy content of the CV is zero.

- This equation states: the net rate of energy transfer to a CV by heat and work transfers during steady flow is equal to the difference between the rates of outgoing and incoming energy flows with mass.

Energy Analysis of Steady Flows



- For single-stream devices, mass flow rate is constant.

$$q_{net,in} + w_{shaft,net,in} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g\left(z_2 - z_1\right)$$

$$w_{shaft,net,in} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + \left(u_2 - u_1 - q_{net,in}\right)$$

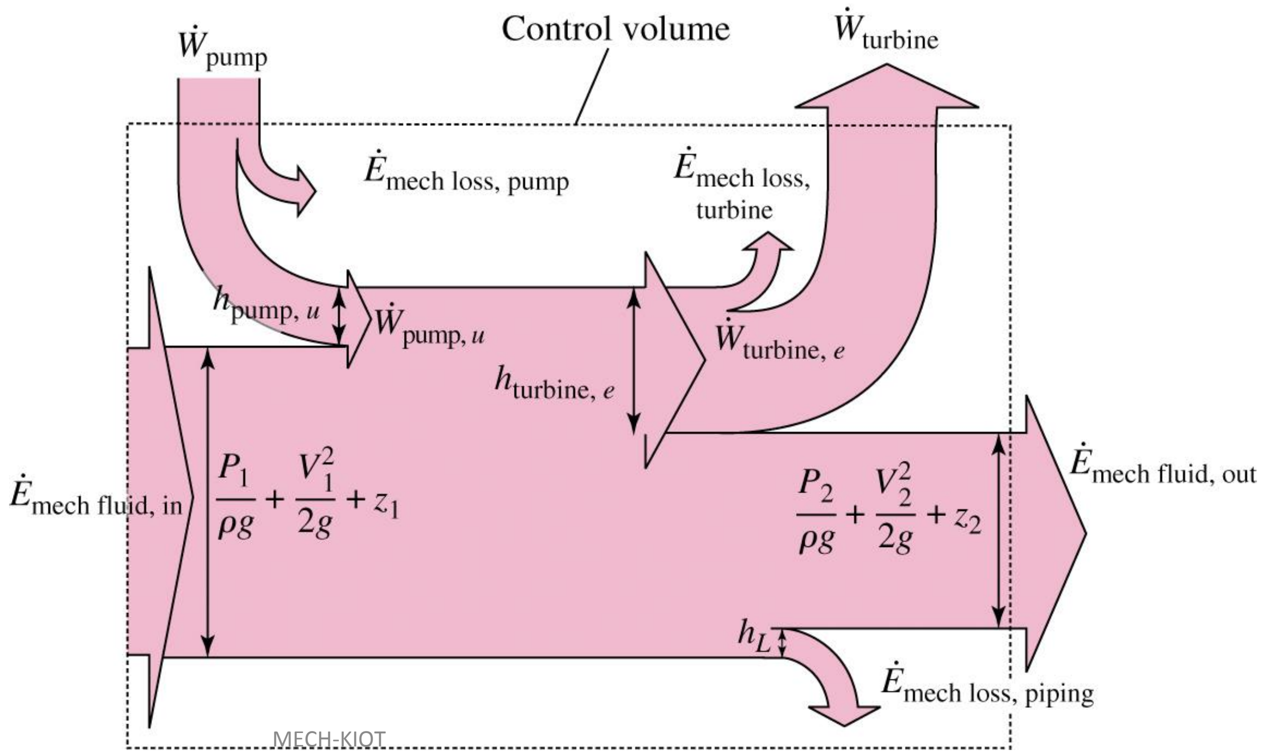
$$\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 + w_{pump} = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + w_{turbine} + e_{mech,loss}$$

Energy Analysis of Steady Flows

Divide by g to get each term in units of length

$$\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 + h_{pump} = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2 + h_{turbine} + h_L$$

Magnitude of each term is now expressed as an equivalent column height of fluid, i.e., Head



Momentum Analysis of Flow Systems

Chapter 6

Introduction

- Fluid flow problems can be analyzed using one of three basic approaches: differential, experimental, and integral (or control volume).

- In previous chapter, control volume forms of the mass and energy equation were developed and used.

- In this chapter, we complete control volume analysis by presenting the linear momentum equation and angular momentum equations.

  - Review Newton's laws and conservation relations for momentum.
  - Use RTT to develop linear and angular momentum equations for control volumes.
  - Use these equations to determine forces and torques acting on the CV.

Objectives

After completing this chapter, you should be able to

- Identify the various kinds of forces and moments acting on a control volume.
- Use control volume analysis to determine the forces associated with fluid flow.
- Use control volume analysis to determine the moments caused by fluid flow and the torque transmitted.

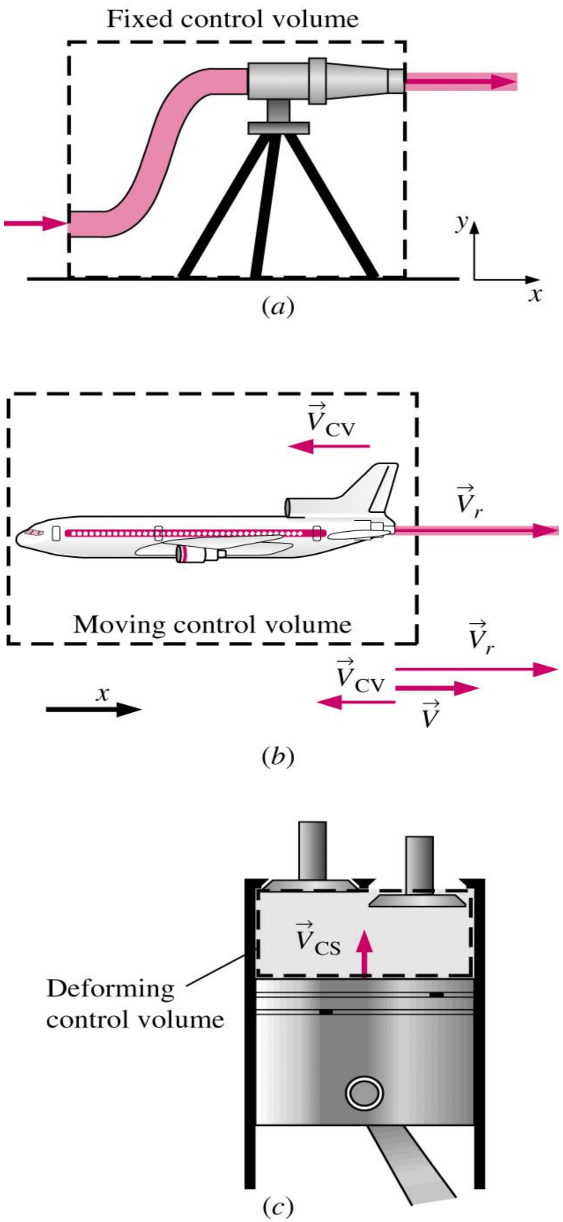
Newton's Laws

- Newton's laws are relations between motions of bodies and the forces acting on them.
  - First law: a body at rest remains at rest, and a body in motion remains in motion at the same velocity in a straight path when the net force acting on it is zero.
  - Second law: the acceleration of a body is proportional to the net force acting on it and is inversely proportional to its mass.

$$\vec{F} = m\vec{a} = m\frac{d\vec{V}}{dt} = \frac{d\left(m\vec{V}\right)}{dt}$$

  - Third law: when a body exerts a force on a second body, the second body exerts an equal and opposite force on the first.

Choosing a Control Volume



- CV is arbitrarily chosen by fluid dynamicist, however, selection of CV can either simplify or complicate analysis.
  - Clearly define all boundaries. Analysis is often simplified if CS is normal to flow direction.
  - Clearly identify all fluxes crossing the CS.
  - Clearly identify forces and torques of interest acting on the CV and CS.
- Fixed, moving, and deforming control volumes.
  - For moving CV, use relative velocity,

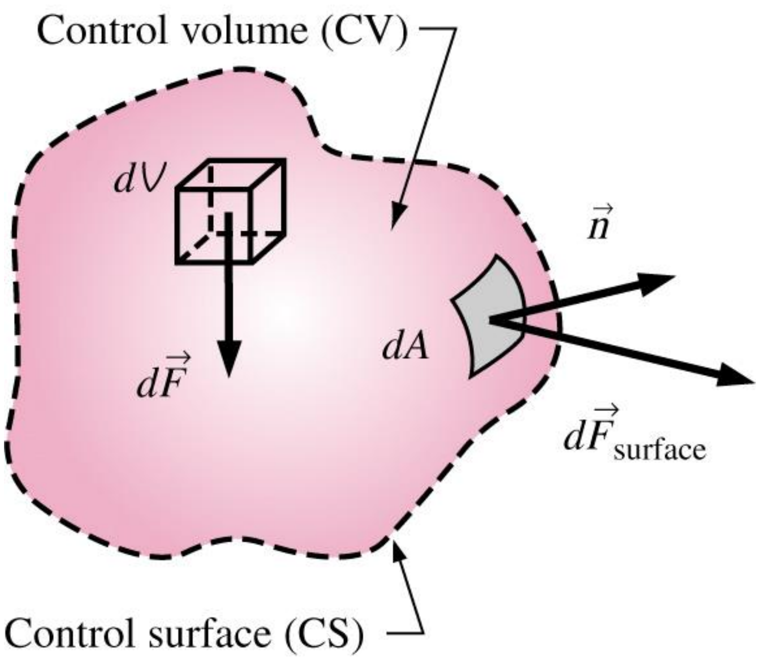
$$\vec{V}_r = \vec{V} - \vec{V}_{CV}$$

  - For deforming CV, use relative velocity all deforming control surfaces,

$$\vec{V}_r = \vec{V} - \vec{V}_{CS}$$

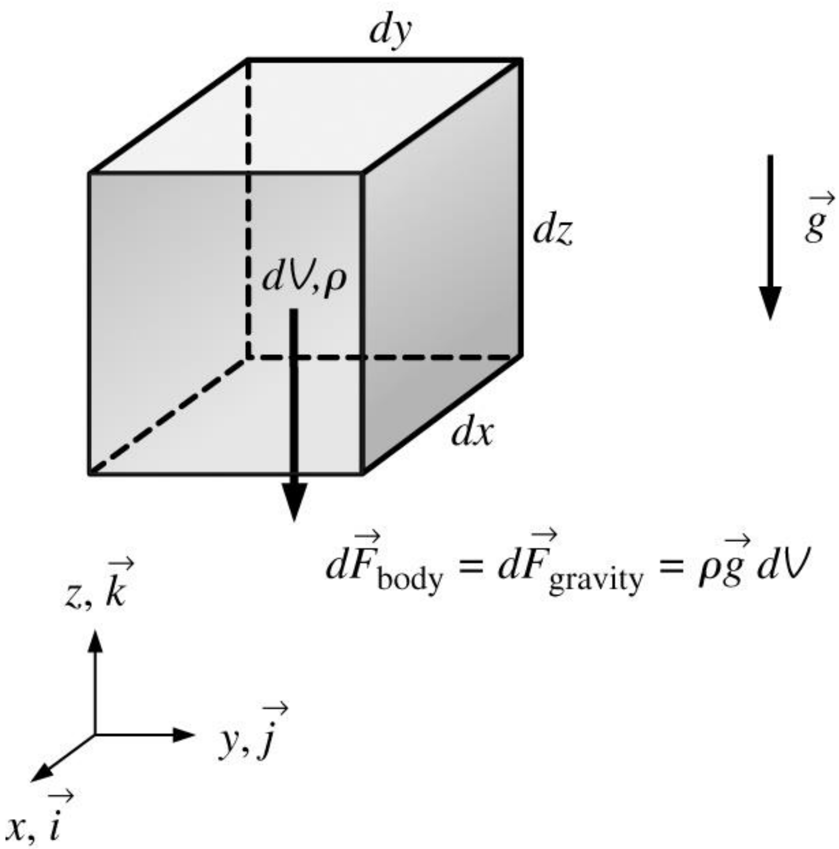
Forces Acting on a CV

- Forces acting on CV consist of body forces that act throughout the entire body of the CV (such as gravity, electric, and magnetic forces) and surface forces that act on the control surface (such as pressure and viscous forces, and reaction forces at points of contact).



- Body forces act on each volumetric portion dV of the CV.
- Surface forces act on each portion dA of the CS.

Body Forces



- The most common body force is gravity, which exerts a downward force on every differential element of the CV

The different body force

$$d\vec{F}_{body} = d\vec{F}_{gravity} = \rho\vec{g}d\mathcal{V}$$

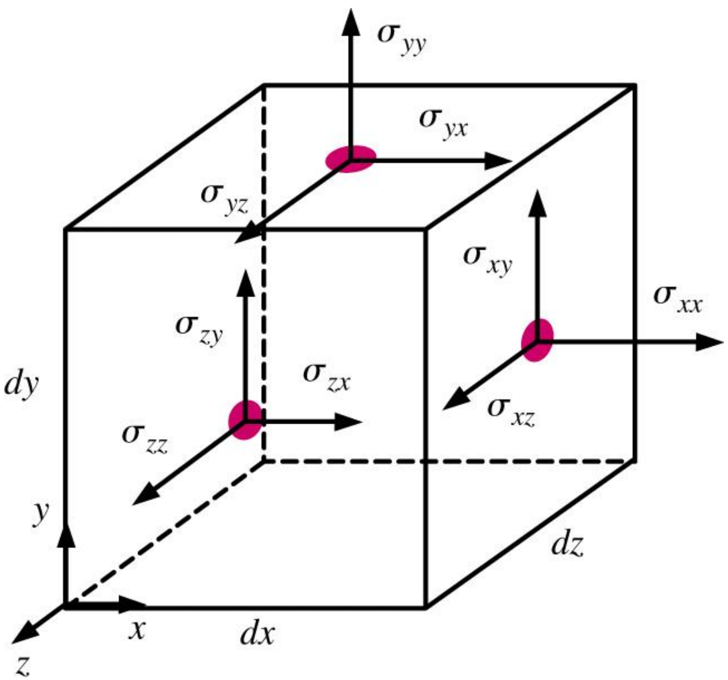
- Typical convention is that $\vec{g}$ acts in the negative z-direction,

$$\vec{g} = -g\vec{k}$$

- Total body force acting on CV

$$\sum \vec{F}_{body} = \int_{CV} \rho\vec{g}d\mathcal{V} = m_{CV}\vec{g}$$

Surface Forces



$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$$

- Surface forces are not as simple to analyze since they include both normal and tangential components
- Diagonal components σ_{xx}, σ_{yy}, σ_{zz} are called normal stresses and are due to pressure and viscous stresses
- Off-diagonal components σ_{xy}, σ_{xz}, etc., are called shear stresses and are due solely to viscous stresses
- Total surface force acting on CS

$$\sum \vec{F}_{surface} = \int_{CS} \sigma_{ij} \cdot \vec{n} dA$$

Body and Surface Forces

$$\sum \vec{F} = \underbrace{\sum \vec{F}_{gravity}}_{\text{body force}} + \underbrace{\sum \vec{F}_{pressure} + \sum \vec{F}_{viscous} + \sum \vec{F}_{other}}_{\text{surface forces}}$$

Linear Momentum Equation

- Newton's second law for a system of mass m subjected to a force $\vec{F}$ is expressed as

$$\vec{F} = m\vec{a} = m\frac{d\vec{V}}{dt} = \frac{d}{dt}\left(m\vec{V}\right)$$

- Use RTT with $b = \vec{V}$ and $B = m\vec{V}$ to shift from system formulation to the control volume formulation

$$\frac{d\left(m\vec{V}\right)_{sys}}{dt} = \frac{d}{dt}\int_{CV}\rho\vec{V}\,d\mathcal{V} + \int_{CS}\rho\vec{V}\left(\vec{V}_r\cdot\vec{n}\right)\,dA$$

$$\sum\vec{F} = \frac{d}{dt}\int_{CV}\rho\vec{V}\,d\mathcal{V} + \int_{CS}\rho\vec{V}\left(\vec{V}_r\cdot\vec{n}\right)\,dA$$

Angular Momentum

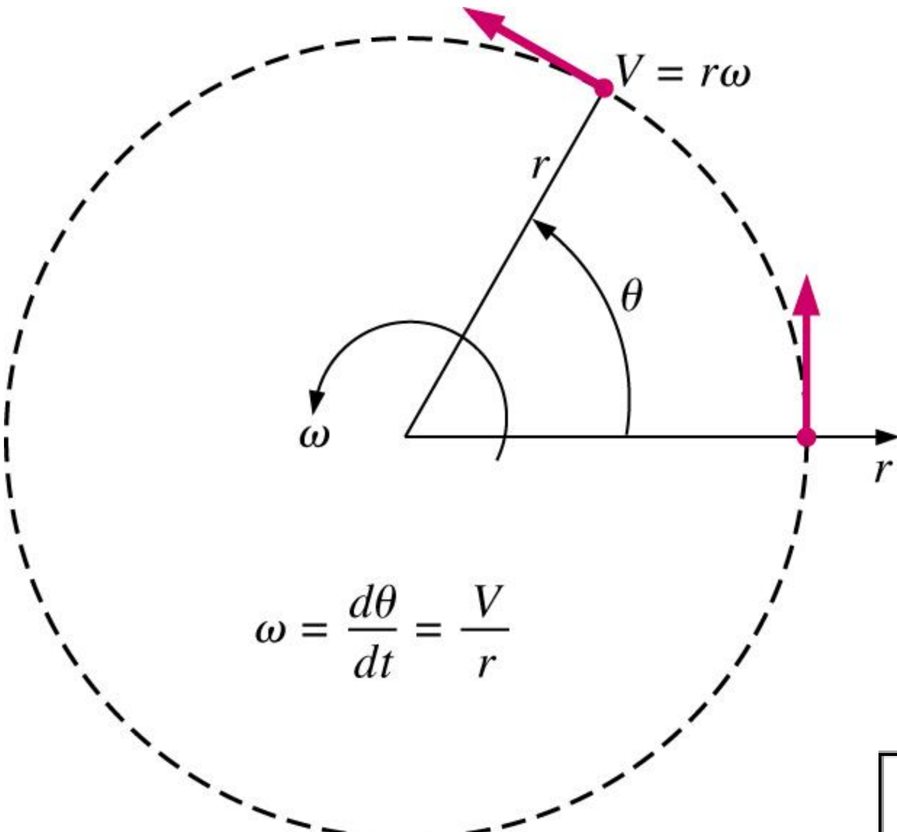
- Motion of a rigid body can be considered to be the combination of
  - the translational motion of its center of mass (U_x, U_y, U_z)
  - the rotational motion about its center of mass (ω_x, ω_y, ω_z)

- Translational motion can be analyzed with linear momentum equation.

- Rotational motion is analyzed with angular momentum equation.

- Together, the body motion can be described as a 6-degree-of-freedom (6DOF) system.

Review of Rotational Motion



Angular velocity ω is the angular distance θ traveled per unit time, and angular acceleration α is the rate of change of angular velocity.

$$\omega = \frac{d\theta}{dt} = \frac{d(lr)}{dt} = \frac{1}{r}\frac{dl}{dt} = \frac{V}{r}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \frac{1}{r}\frac{dV}{dt} = \frac{a_t}{r}$$

$$\boxed{V = r\omega \text{ and } a_t = r\alpha}$$

Review of Angular Momentum

- Moment of a force:

$$\vec{M} = \vec{r} \times \vec{F}$$

- Moment of momentum:

$$\vec{H} = \vec{r} \times m\vec{V}$$

- For a system:

$$\vec{H}_{sys} = \int_{sys} (\vec{r} \times \vec{V})\rho \, d\mathcal{V}$$

$$\frac{d\vec{H}_{sys}}{dt} = \frac{d}{dt} \int_{sys} (\vec{r} \times \vec{V})\rho \, d\mathcal{V}$$

- Therefore, the angular momentum equation can be written as:

$$\sum \vec{M} = \frac{d\vec{H}_{sys}}{dt}$$

- To derive angular momentum for a CV, use RTT with $B = \vec{H}$ and $\beta = \vec{r} \times \vec{V}$

Angular Momentum Equation for a CV

- General form

$$\sum \vec{M} = \frac{d}{dt}\int_{CV} (\vec{r} \times \vec{V})\rho \, d\mathcal{V} + \int_{CS} (\vec{r} \times \vec{V})\rho \left(\vec{V}_r \cdot \vec{n}\right) \, dA$$

- Approximate form using average properties at inlets and outlets

$$\sum \vec{M} = \frac{d}{dt}\int_{CV} (\vec{r} \times \vec{V})\rho \, d\mathcal{V} + \sum_{out} \vec{r} \times \dot{m}\vec{V} - \sum_{in} \vec{r} \times \dot{m}\vec{V}$$

- Steadv flow

$$\sum \vec{M} = +\sum_{out} \vec{r} \times \dot{m}\vec{V} - \sum_{in} \vec{r} \times \dot{m}\vec{V}$$

Dimensional Analysis and Modeling

Chapter 7

Objectives

- Understand dimensions, units, and dimensional homogeneity
- Understand benefits of dimensional analysis
- Know how to use the method of repeating variables
- Understand the concept of similarity and how to apply it to experimental modeling

Dimensions and Units

- Review
  - Dimension: Measure of a physical quantity, e.g., length, time, mass
  - Units: Assignment of a number to a dimension, e.g., (m), (sec), (kg)
  - 7 Primary Dimensions:
    - Mass M (kg)
    - Length L (m)
    - Time T (sec)
    - Temperature θ (K)
    - Current I (A)
    - Amount of Light C (cd)
    - Amount of matter N (mol)

Property	Dimensions	Property	Dimensions
Acceleration	Lt^{-2}	Momentum	MLt^{-1}
Angle	Dimensionless	Power	ML^2t^{-3}
Angular momentum	ML^2t^{-2}	Pressure	$ML^{-1}t^{-2}$
Angular velocity	t^{-1}	Specific heat	$L^2t^{-2}T^{-1}$
Area	L^2	Specific weight	$ML^{-2}t^{-2}$
Density	ML^{-3}	Strain	Dimensionless
Energy	ML^2t^{-2}	Stress	$ML^{-1}t^{-2}$
Force	MLt^{-2}	Surface tension	Mt^{-2}
Frequency	t^{-1}	Temperature	T
Heat	ML^2t^{-2}	Time	t
Length	L	Torque	ML^2t^{-2}
Mass	M	Velocity	Lt^{-1}
Modulus of elasticity	$ML^{-1}t^{-2}$	Viscosity (dynamic)	$ML^{-1}t^{-1}$
Moment of a force	ML^2t^{-2}	Viscosity (kinematic)	L^2t^{-1}
Moment of inertia (area)	L^4	Volume	L^3
Moment of inertia (mass)	ML^2	Work	ML^2t^{-2}

Quantity	Symbol	$MLT\Theta$
Length	L	L
Area	A	L^2
Volume	$\mathcal{V}$	L^3
Velocity	V	LT^{-1}
Acceleration	dV/dt	LT^{-2}
Speed of sound	a	LT^{-1}
Volume flow	Q	L^3T^{-1}
Mass flow	$\dot{m}$	MT^{-1}
Pressure, stress	p, σ	$ML^{-1}T^{-2}$
Strain rate	$\dot{\epsilon}$	T^{-1}
Angle	θ	None
Angular velocity	ω	T^{-1}
Viscosity	μ	$ML^{-1}T^{-1}$
Kinematic viscosity	ν	L^2T^{-1}
Surface tension	Υ	MT^{-2}
Force	F	MLT^{-2}
Moment, torque	M	ML^2T^{-2}
Power	P	ML^2T^{-3}
Work, energy	W, E	ML^2T^{-2}
Density	ρ	ML^{-3}
Temperature	T	Θ
Specific heat	c_p, c_v	$L^2T^{-2}\Theta^{-1}$
Specific weight	γ	$ML^{-2}T^{-2}$
Thermal conductivity	k	$MLT^{-3}\Theta^{-1}$
Expansion coefficient	β	Θ^{-1}

Dimensions and Units

- All non-primary dimensions can be formed by a combination of the 7 primary dimensions

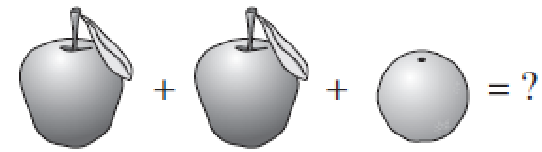
- Examples

  - {Velocity} = {Length/Time} = {L/T}

  - {Force} = {Mass Length/Time} = $\{ML/T^2\}$

Dimensional Homogeneity

Law of dimensional homogeneity (DH): every additive term in an equation must have the same dimensions



You can't add apples and oranges!

Example: Bernoulli equation

$$p + \frac{1}{2}\rho V^2 + \rho g z = C$$

- {p} = {force/area}={mass x length/time^2 x 1/length^2} = {M/(T^{2}L)}
- {1/2ρV^2} = {mass/length^3 x (length/time)^2} = {M/(T^{2}L)}
- {ρgz} = {mass/length^3 x length/time^2 x length} ={M/(T^{2}L)}

Nondimensionalization of Equations

- Given the law of DH, if we divide each term in the equation by a collection of variables and constants that have the same dimensions, the equation is rendered non-dimensional

- In the process of nondimensionalizing an equation, nondimensional parameters often appear, e.g., Reynolds number and Froude number

Nondimensionalization of Equations

- To nondimensionalize, for example, the Bernoulli equation, the first step is to list primary dimensions of all dimensional variables and constants

$$p + \frac{1}{2}\rho V^2 + \rho g z = C$$

- $\{p\} = \{M/(T^2L)\}$ $\{\rho\} = \{M/L^3\}$ $\{V\} = \{L/T\}$
- $\{g\} = \{L/T^2\}$ $\{z\} = \{L\}$

- Next, we need to select Scaling Parameters. For this example, select L, U_O, ρ_O

Nondimensionalization of Equations

- By inspection, nondimensionalize all variables with scaling pa

$$p^\star = \frac{p}{\rho_0 U_0^2} \qquad \rho^\star = \frac{\rho}{\rho_0} \qquad V^\star = \frac{V}{U_0}$$

$$g^\star = \frac{gL}{U_0^2} \qquad z^\star = \frac{z}{L}$$

$$\rho_0 U_0^2 p^\star + \frac{1}{2}\rho_0 \rho^\star \left(U_0^2 V^{\star 2}\right) + \rho_0 \rho^\star g^\star U_0^2 z^\star = C$$

Nondimensionalization of Equations

- Divide by $\rho_0 U_0^2$ and set $\rho^* = 1$ (incompressible flow)

$$p^\star + \frac{1}{2}V^{\star 2} + g^\star z^\star = \frac{C}{\rho_0 U_0^2} = C^\star$$

- Since $g^* z^* = 1/Fr^2$, where

Froude number $\quad \mathrm{Fr} = \frac{V}{\sqrt{gL}} \left(\text{sometimes } \frac{V^2}{gL} \right) \quad \frac{\text{Inertial force}}{\text{Gravitational force}}$

$$Fr = \frac{U_0}{\sqrt{gL}}$$

$$p^\star + \frac{1}{2}V^{\star 2} + \frac{1}{Fr^2} z^\star = C^\star$$

Nondimensionalization of Equations

Advantages of non-dimensionalization

- Increases insight about key parameters
- Decreases number of parameters in the problem

  Easier communication

  Fewer experiments

  Fewer simulations

- Extrapolation of results to untested conditions

Dimensionless numbers

S. N	Dimensionless numbers	Symbol	Group of Variables	Field of application
1	Reynolds number (IF/VF)	Re	$\frac{\rho V L}{\mu}$	Laminar viscous flow in confined passages (where viscous effects are significant)
2	Froude number (IF/GF)	Fr	$\frac{V}{\sqrt{Lg}}$	Free surface flows (where gravity effects are important)
3	Euler number (IF/PF)	Eu	$\frac{V}{\sqrt{p/\rho}}$	Conduit flow (where pressure variations are significant)
4	Weber number (IF/STF)	We	$\frac{V}{\sqrt{\sigma/\rho L}}$	Small surfaces waves, capillary and sheet flow (where surface tension is important)
5	Mach number (IF/EF)	M	$\frac{V}{\sqrt{K/\rho}}$	High speed flow (where compressibility effects are significant)

Dimensional Analysis

- Nondimensionalization of an equation is useful only when the equation is known!
- In many real-world flows, the equations are either unknown or too difficult to solve.
  - Experimentation is the only method of obtaining reliable information
  - In most experiments, geometrically-scaled models are used (time and money).
  - Experimental conditions and results must be properly scaled so that results are meaningful for the full-scale prototype.
  - We need to introduce a powerful technique called Dimensional Analysis

Dimensional Analysis and Similarity

Primary purposes of dimensional analysis

- To generate nondimensional parameters that help in the design of experiments (physical and/or numerical) and in reporting of results.
- To obtain scaling laws so that prototype performance can be predicted from model performance.
- To predict trends in the relationship between parameters.

Dimensional Analysis and Similarity

Various methods

1) Buckingham's Pi theorem

2) Rayleigh's method

3) Bridgman's method

4) Matrix-Tensor method

Buckingham's Pi theorem

- If there are n variables (dependent and independent) in a dimensionally homogeneous equation and if these variables contain m fundamental dimensions (such as M,L,T,θ), the variables are arranged into (n-m) dimensionless terms.
- These dimensionless terms are called as Pi theorem

$$\Pi = f(\Pi_1, \Pi_2, \ldots\ldots, \Pi_n)$$

The Method Of Repeating Variables And The Buckingham Pi Theorem

The Method of Repeating Variables

Step 1: List the parameters in the problem and count their total number n.

Step 2: List the primary dimensions of each of the n parameters.

Step 3: Set the *reduction* j as the number of primary dimensions. Calculate k, the expected number of Π's,

$$k = n - j$$

Step 4: Choose j *repeating parameters*.

Step 5: Construct the k Π's, and manipulate as necessary.

Step 6: Write the final functional relationship and check your algebra.

Guidelines for choosing repeating parameters in step 4 of the method of repeating variables

Guideline

1. Never pick the *dependent* variable. Otherwise, it may appear in all the Π's, which is undesirable.
2. The chosen repeating parameters must not *by themselves* be able to form a dimensionless group. Otherwise, it would be impossible to generate the rest of the Π's.
3. The chosen repeating parameters must represent *all* the primary dimensions in the problem.

4. Never pick parameters that are already dimensionless. These are Π's already, all by themselves.
5. Never pick two parameters with the *same* dimensions or with dimensions that differ by only an exponent.
6. Whenever possible, choose dimensional constants over dimensional variables so that only *one* Π contains the dimensional variable.
7. Pick common parameters since they may appear in each of the Π's.

Guidelines for manipulation of the Π's resulting from the method of repeating variables

Guideline

1. We may impose a constant (dimensionless) exponent on a Π or perform a functional operation on a Π.
2. We may multiply a Π by a pure (dimensionless) constant.

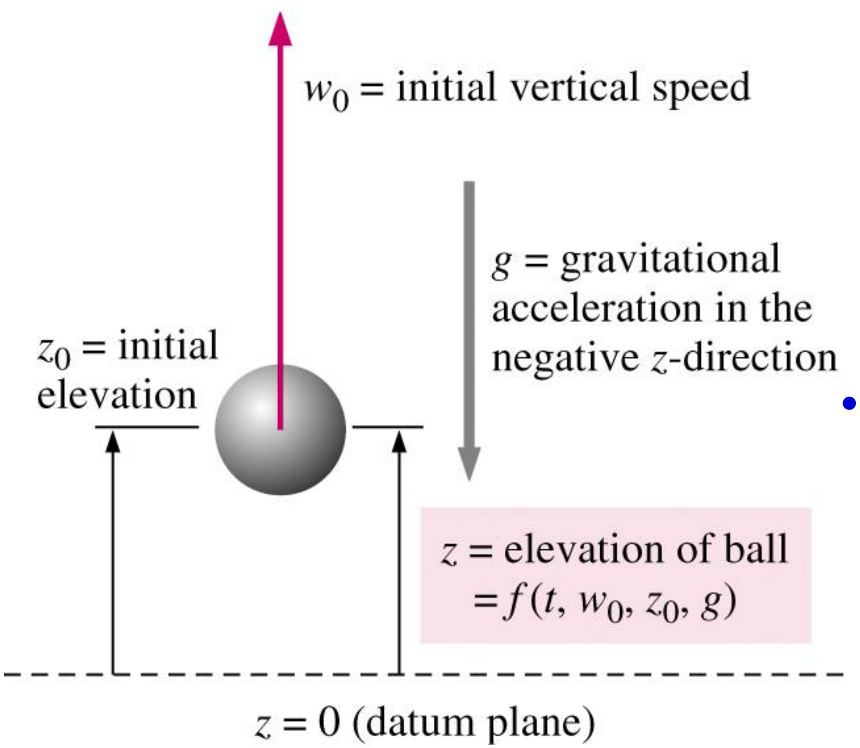
3. We may form a product (or quotient) of any Π with any other Π in the problem to replace one of the Π's.

4. We may use any of guidelines 1 to 3 in combination.

5. We may substitute a dimensional parameter in the Π with other parameter(s) of the same dimensions.

Example

Ball Falling in a Vacuum



- Step 1: List relevant parameters. $z=f(t,w_0,z_0,g) \Rightarrow n=5$

- Step 2: Primary dimensions of each parameter

z	t	w_0	z_0	g
$\{L^1\}$	$\{t^1\}$	$\{L^1t^{-1}\}$	$\{L^1\}$	$\{L^1t^{-2}\}$

- Step 3: As a first guess, reduction j is set to 2 which is the number of primary dimensions (L and t). Number of expected Π's is $k=n-j=5-2=3$

- Step 4: Choose repeating variables w_0 and z_0

Example, continued

- Step 5: Combine repeating parameters into products with each of the remaining parameters, one at a time, to create the Π's.
- $\Pi_1 = z\, w_O^{a1} z_O^{b1}$
  - a1 and b1 are constant exponents which must be determined.
  - Use the primary dimensions identified in Step 2 and solve for a_1 and b_1.

$$\{\Pi_1\} = \{L^0 t^0\} = \{z w_0^{a_1} z_0^{b_1}\} = \{L^1 (L^1 t^{-1})^{a_1} L^{b_1}\}$$

  - Time equation:

$$\{t^0\} = \{t^{-a_1}\} \rightarrow 0 = -a_1 \rightarrow a_1 = 0$$

  - Length equation:

$$\{L^0\} = \{L^1 L^{a_1} L^{b_1}\} \rightarrow 0 = 1 + a_1 + b_1 \rightarrow b_1 = -1 - a_1 \rightarrow b_1 = -1$$

  - This results in

$$\Pi_1 = z w_0^0 z_0^{-1} = \frac{z}{z_0}$$

Example, continued

- Step 5: continued
  - Repeat process for Π_2 by combining repeating parameters with t
  - $\Pi_2 = t\, w_0^{a2} z_0^{b2}$

$$\{\Pi_2\} = \{L^0 t^0\} = \{t w_0^{a_2} z_0^{b_2}\} = \{t^1 (L^1 t^{-1})^{a_2} L^{b_2}\}$$

  - Time equation:

$$\{t^0\} = \{t^1 t^{-a_2}\} \rightarrow 0 = 1 - a_2 \rightarrow a_2 = 1$$

  - Length equation:

$$\{L^0\} = \{L^{a_2} L^{b_2}\} \rightarrow 0 = a_2 + b_2 \rightarrow b_2 = -a_2 \rightarrow b_2 = -1$$

  - This results in

$$\Pi_2 = t w_0^1 z_0^{-1} = \frac{w_0 t}{z_0}$$

Example, continued

- Step 5: continued
  - Repeat process for Π_3 by combining repeating parameters with g
  - $\Pi_3 = g\, w_0^{a3} z_0^{b3}$

$$\{\Pi_3\} = \{L^0 t^0\} = \{g w_0^{a_3} z_0^{b_3}\} = \{L^1 t^{-2} (L^1 t^{-1})^{a_3} L^{b_3}\}$$

Time equation:

$$\{t^0\} = \{t^{-2} t^{-a_3}\} \rightarrow 0 = -2 - a_3 \rightarrow a_3 = -2$$

Length equation:

$$\{L^0\} = \{L^1 L^{a_3} L^{b_3}\} \rightarrow 0 = 1 + a_3 + b_3 \rightarrow b_3 = -1 - a_3 \rightarrow b_3 = 1$$

$$\Pi_3 = g w_0^{-2} z_0^1 = \frac{g z_0}{w_0^2}$$

$$\Pi_{3,modified} = \left(\frac{g z_0}{w_0^2}\right)^{-1/2} = \frac{w_0}{\sqrt{g z_0}} = Fr$$

Example, continued

- Step 6:
  - Double check that the Π's are dimensionless.
  - Write the functional relationship between Π's

$$\Pi_1 = f(\Pi_2, \Pi_3) \rightarrow \frac{z}{z_0} = f\left(\frac{w_0 t}{z_0}, \frac{w_0}{\sqrt{g z_0}}\right)$$

  - Or, in terms of nondimensional variables

$$z^* = f(t^*, Fr)$$

- Overall conclusion: Method of repeating variables properly predicts the functional relationship between dimensionless groups.
- However, the method cannot predict the exact mathematical form of the equation.

Detailed description of the six steps that comprise the *method of repeating variables**

Step 1 List the parameters (dimensional variables, nondimensional variables, and dimensional constants) and count them. Let n be the total number of parameters in the problem, including the dependent variable. Make sure that any listed independent parameter is indeed independent of the others, i.e., it cannot be expressed in terms of them. (E.g., don't include radius r and area $A = \pi r^2$, since r and A are *not* independent.)

Step 2 List the primary dimensions for each of the n parameters.

Step 3 Guess the **reduction** j. As a first guess, set j equal to the number of primary dimensions represented in the problem. The expected number of Π's (k) is equal to n minus j, according to the **Buckingham Pi theorem**,

The Buckingham Pi theorem: $$k = n - j \qquad (7\text{–}14)$$

If at this step or during any subsequent step, the analysis does not work out, verify that you have included enough parameters in step 1. Otherwise, go back and *reduce j by one* and try again.

Step 4 Choose j **repeating parameters** that will be used to construct each Π. Since the repeating parameters have the potential to appear in each Π, be sure to choose them *wisely* (Table 7–3).

Step 5 Generate the Π's one at a time by grouping the j repeating parameters with one of the remaining parameters, forcing the product to be dimensionless. In this way, construct all k Π's. By convention the first Π, designated as Π_1, is the *dependent* Π (the one on the left side of the list). Manipulate the Π's as necessary to achieve established dimensionless groups (Table 7–5).

Step 6 Check that all the Π's are indeed dimensionless. Write the final functional relationship in the form of Eq. 7–11.

Guidelines for choosing *repeating parameters* in step 4 of the method of repeating variables*

Guideline	Comments and Application to Present Problem
1. Never pick the *dependent* variable. Otherwise, it may appear in all the Π's, which is undesirable.	In the present problem we cannot choose z, but we must choose from among the remaining four parameters. Therefore, we must choose two of the following parameters: t, w_0, z_0, and g.
2. The chosen repeating parameters must not *by themselves* be able to form a dimensionless group. Otherwise, it would be impossible to generate the rest of the Π's.	In the present problem, any two of the independent parameters would be valid according to this guideline. For illustrative purposes, however, suppose we have to pick three instead of two repeating parameters. We could not, for example, choose t, w_0, and z_0, because these can form a Π all by themselves (tw_0/z_0).
3. The chosen repeating parameters must represent *all* the primary dimensions in the problem.	Suppose for example that there were *three* primary dimensions (m, L, and t) and *two* repeating parameters were to be chosen. You could not choose, say, a length and a time, since primary dimension mass would not be represented in the dimensions of the repeating parameters. An appropriate choice would be a density and a time, which together represent all three primary dimensions in the problem.
4. Never pick parameters that are already dimensionless. These are Π's already, all by themselves.	Suppose an angle θ were one of the independent parameters. We could not choose θ as a repeating parameter since angles have no dimensions (radian and degree are dimensionless units). In such a case, one of the Π's is already known, namely θ.
5. Never pick two parameters with the *same* dimensions or with dimensions that differ by only an exponent.	In the present problem, two of the parameters, z and z_0, have the same dimensions (length). We cannot choose both of these parameters. (Note that dependent variable z has already been eliminated by guideline 1.) Suppose one parameter has dimensions of length and another parameter has dimensions of volume. In dimensional analysis, volume contains only one primary dimension (length) and *is not dimensionally distinct from length*—we cannot choose both of these parameters.

6. Whenever possible, choose dimensional constants over dimensional variables so that only *one* Π contains the dimensional variable.	If we choose time t as a repeating parameter in the present problem, it would appear in all three Π's. While this would not be *wrong*, it would not be *wise* since we know that ultimately we want some nondimensional height as a function of some nondimensional time and other nondimensional parameter(s). From the original four independent parameters, this restricts us to w_0, z_0, and g.
7. Pick common parameters since they may appear in each of the Π's.	In fluid flow problems we generally pick a length, a velocity, and a mass or density (Fig. 7–25). It is unwise to pick less common parameters like viscosity μ or surface tension σ_s, since we would in general not want μ or σ_s to appear in each of the Π's. In the present problem, w_0 and z_0 are wiser choices than g.
8. Pick simple parameters over complex parameters whenever possible.	It is better to pick parameters with only one or two basic dimensions (e.g., a length, a time, a mass, or a velocity) instead of parameters that are composed of several basic dimensions (e.g., an energy or a pressure).

Guidelines for manipulation of the Π's resulting from the method of repeating variables.*

Guideline	Comments and Application to Present Problem
1. We may impose a constant (dimensionless) exponent on a Π or perform a functional operation on a Π.	We can raise a Π to any exponent n (changing it to Π^n) without changing the dimensionless stature of the Π. For example, in the present problem, we imposed an exponent of $-\frac{1}{2}$ on Π_3. Similarly we can perform the functional operation sin(Π), exp(Π), etc., without influencing the dimensions of the Π.
2. We may multiply a Π by a pure (dimensionless) constant.	Sometimes dimensionless factors of $\frac{1}{2}$, 2, 4, etc., are included in a Π for convenience. This is perfectly okay since such factors do not influence the dimensions of the Π.
3. We may form a product (or quotient) of any Π with any other Π in the problem to replace one of the Π's.	We could replace Π_3 by $\Pi_3\Pi_1$, Π_3/Π_2, etc. Sometimes such manipulation is necessary to convert our Π into an established Π. In many cases, the established Π would have been produced if we would have chosen different repeating parameters.
4. We may use any of guidelines 1 to 3 in combination.	In general, we can replace any Π with some new Π such as $A\Pi_3^B \sin(\Pi_1^C)$, where A, B, and C are pure constants.
5. We may substitute a dimensional parameter in the Π with other parameter(s) of the same dimensions.	For example, the Π may contain the square of a length or the cube of a length, for which we may substitute a known area or volume, respectively, in order to make the Π agree with established conventions.

Some common established nondimensional parameters or Π's encountered in fluid mechanics and heat transfer*

Name	Definition	Ratio of Significance
Archimedes number	$\text{Ar} = \dfrac{\rho_s g L^3}{\mu^2}(\rho_s - \rho)$	$\dfrac{\text{Gravitational force}}{\text{Viscous force}}$
Aspect ratio	$\text{AR} = \dfrac{L}{W}$ or $\dfrac{L}{D}$	$\dfrac{\text{Length}}{\text{Width}}$ or $\dfrac{\text{Length}}{\text{Diameter}}$
Biot number	$\text{Bi} = \dfrac{hL}{k}$	$\dfrac{\text{Surface thermal resistance}}{\text{Internal thermal resistance}}$
Bond number	$\text{Bo} = \dfrac{g(\rho_f - \rho_v)L^2}{\sigma_s}$	$\dfrac{\text{Gravitational force}}{\text{Surface tension force}}$
Cavitation number	$\text{Ca (sometimes } \sigma_c) = \dfrac{P - P_v}{\rho V^2}$ $\left(\text{sometimes } \dfrac{2(P - P_v)}{\rho V^2}\right)$	$\dfrac{\text{Pressure} - \text{Vapor pressure}}{\text{Inertial pressure}}$

Darcy friction factor	$f = \frac{8\tau_w}{\rho V^2}$	$\frac{\text{Wall friction force}}{\text{Inertial force}}$
Drag coefficient	$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$	$\frac{\text{Drag force}}{\text{Dynamic force}}$
Eckert number	$\text{Ec} = \frac{V^2}{c_P T}$	$\frac{\text{Kinetic energy}}{\text{Enthalpy}}$
Euler number	$\text{Eu} = \frac{\Delta P}{\rho V^2} \left(\text{sometimes } \frac{\Delta P}{\frac{1}{2}\rho V^2}\right)$	$\frac{\text{Pressure difference}}{\text{Dynamic pressure}}$
Fanning friction factor	$C_f = \frac{2\tau_w}{\rho V^2}$	$\frac{\text{Wall friction force}}{\text{Inertial force}}$
Fourier number	$\text{Fo (sometimes } \tau) = \frac{\alpha t}{L^2}$	$\frac{\text{Physical time}}{\text{Thermal diffusion time}}$

Froude number	$\mathrm{Fr} = \frac{V}{\sqrt{gL}} \left(\text{sometimes } \frac{V^2}{gL}\right)$	$\frac{\text{Inertial force}}{\text{Gravitational force}}$
Grashof number	$\mathrm{Gr} = \frac{g\beta \lvert\Delta T\rvert L^3 \rho^2}{\mu^2}$	$\frac{\text{Buoyancy force}}{\text{Viscous force}}$
Jakob number	$\mathrm{Ja} = \frac{c_p(T - T_{\mathrm{sat}})}{h_{fg}}$	$\frac{\text{Sensible energy}}{\text{Latent energy}}$
Knudsen number	$\mathrm{Kn} = \frac{\lambda}{L}$	$\frac{\text{Mean free path length}}{\text{Characteristic length}}$
Lewis number	$\mathrm{Le} = \frac{k}{\rho c_p D_{AB}} = \frac{\alpha}{D_{AB}}$	$\frac{\text{Thermal diffusion}}{\text{Species diffusion}}$
Lift coefficient	$C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A}$	$\frac{\text{Lift force}}{\text{Dynamic force}}$

Mach number	$\text{Ma (sometimes } M) = \frac{V}{c}$	$\frac{\text{Flow speed}}{\text{Speed of sound}}$
Nusselt number	$\text{Nu} = \frac{Lh}{k}$	$\frac{\text{Convection heat transfer}}{\text{Conduction heat transfer}}$
Peclet number	$\text{Pe} = \frac{\rho L V c_p}{k} = \frac{LV}{\alpha}$	$\frac{\text{Bulk heat transfer}}{\text{Conduction heat transfer}}$
Power number	$N_P = \frac{\dot{W}}{\rho D^5 \omega^3}$	$\frac{\text{Power}}{\text{Rotational inertia}}$
Prandtl number	$\text{Pr} = \frac{\nu}{\alpha} = \frac{\mu c_p}{k}$	$\frac{\text{Viscous diffusion}}{\text{Thermal diffusion}}$
Pressure coefficient	$C_p = \frac{P - P_\infty}{\frac{1}{2}\rho V^2}$	$\frac{\text{Static pressure difference}}{\text{Dynamic pressure}}$

Pressure coefficient	$C_p = \dfrac{P - P_\infty}{\frac{1}{2}\rho V^2}$	$\dfrac{\text{Static pressure difference}}{\text{Dynamic pressure}}$
Rayleigh number	$\text{Ra} = \dfrac{g\beta \lvert\Delta T\rvert L^3 \rho^2 c_p}{k\mu}$	$\dfrac{\text{Buoyancy force}}{\text{Viscous force}}$
Reynolds number	$\text{Re} = \dfrac{\rho V L}{\mu} = \dfrac{VL}{\nu}$	$\dfrac{\text{Inertial force}}{\text{Viscous force}}$
Richardson number	$\text{Ri} = \dfrac{L^5 g\, \Delta\rho}{\rho \dot{V}^2}$	$\dfrac{\text{Buoyancy force}}{\text{Inertial force}}$
Schmidt number	$\text{Sc} = \dfrac{\mu}{\rho D_{AB}} = \dfrac{\nu}{D_{AB}}$	$\dfrac{\text{Viscous diffusion}}{\text{Species diffusion}}$
Sherwood number	$\text{Sh} = \dfrac{VL}{D_{AB}}$	$\dfrac{\text{Overall mass diffusion}}{\text{Species diffusion}}$

Specific heat ratio	$k \text{ (sometimes } \gamma) = \dfrac{c_p}{c_v}$	$\dfrac{\text{Enthalpy}}{\text{Internal energy}}$
Stanton number	$\text{St} = \dfrac{h}{\rho c_p V}$	$\dfrac{\text{Heat transfer}}{\text{Thermal capacity}}$
Stokes number	$\text{Stk (sometimes St)} = \dfrac{\rho_p D_p^2 V}{18 \mu L}$	$\dfrac{\text{Particle relaxation time}}{\text{Characteristic flow time}}$
Strouhal number	$\text{St (sometimes S or Sr)} = \dfrac{fL}{V}$	$\dfrac{\text{Characteristic flow time}}{\text{Period of oscillation}}$
Weber number	$\text{We} = \dfrac{\rho V^2 L}{\sigma_s}$	$\dfrac{\text{Inertial force}}{\text{Surface tension force}}$

Similarity

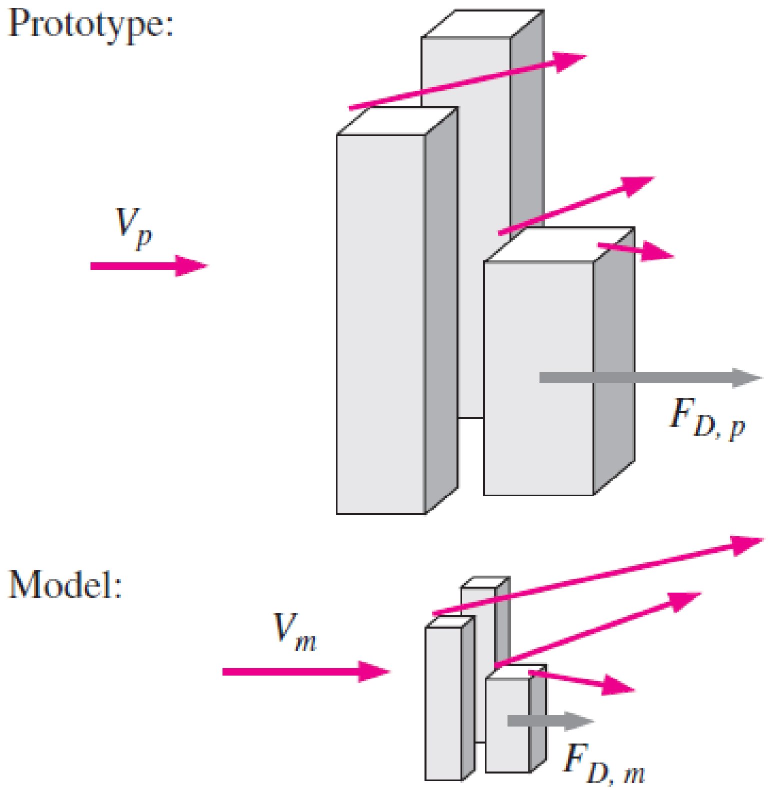
- Geometric Similarity - the model must be the same shape as the prototype. Each dimension must be scaled by the same factor.

- Kinematic Similarity - velocity at any point in the model must be proportional .

- Dynamic Similarity - all forces in the model flow scale by a constant factor to corresponding forces in the prototype flow.

- Complete Similarity is achieved only if all 3 conditions are met.

In a general flow field, complete similarity between a model and prototype is achieved only when there is geometric, kinematic, and dynamic similarity.



Kinematic similarity is achieved when, at all locations, the velocity in the model flow is proportional to that at corresponding locations in the prototype flow, and points in the same direction.

Some Common Variables and Dimensionless Groups in Fluid Mechanics

Variables: Acceleration of gravity, g; Bulk modulus, E_v; Characteristic length, ℓ; Density, ρ; Frequency of oscillating flow, ω; Pressure, p (or Δp); Speed of sound, c; Surface tension, σ; Velocity, V; Viscosity, μ

Dimensionless Groups	Name	Interpretation (Index of Force Ratio Indicated)	Types of Applications
$\frac{\rho V \ell}{\mu}$	Reynolds number, Re	$\frac{\text{inertia force}}{\text{viscous force}}$	Generally of importance in all types of fluid dynamics problems
$\frac{V}{\sqrt{g\ell}}$	Froude number, Fr	$\frac{\text{inertia force}}{\text{gravitational force}}$	Flow with a free surface
$\frac{p}{\rho V^2}$	Euler number, Eu	$\frac{\text{pressure force}}{\text{inertia force}}$	Problems in which pressure, or pressure differences, are of interest
$\frac{\rho V^2}{E_v}$	Cauchy number,[a] Ca	$\frac{\text{inertia force}}{\text{compressibility force}}$	Flows in which the compressibility of the fluid is important
$\frac{V}{c}$	Mach number,[a] Ma	$\frac{\text{inertia force}}{\text{compressibility force}}$	Flows in which the compressibility of the fluid in important
$\frac{\omega \ell}{V}$	Strouhal number, St	$\frac{\text{inertia (local) force}}{\text{inertia (convective) force}}$	Unsteady flow with a characteristic frequency of oscillation
$\frac{\rho V^2 \ell}{\sigma}$	Weber number, We	$\frac{\text{inertia force}}{\text{surface tension force}}$	Problems in which surface tension is important

Incomplete Similarity

It is not always possible to match all the Π's of a model to the corresponding Π's of the prototype, even if we are careful to achieve geometric similarity. This situation is called incomplete similarity.

In such case we need to extrapolate model tests to obtain reasonable full- scale predictions

Flow in Pipes

Chapter 8

Objectives

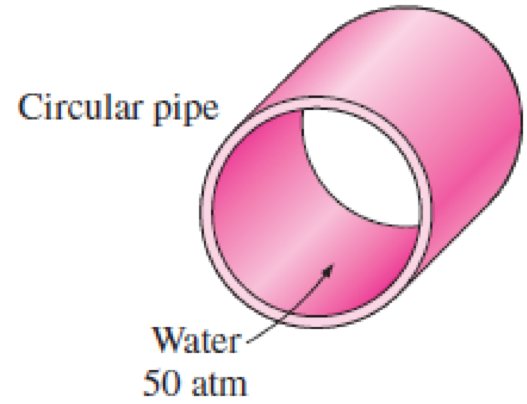
- Have a deeper understanding of laminar and turbulent flow in pipes and the analysis of fully developed flow.

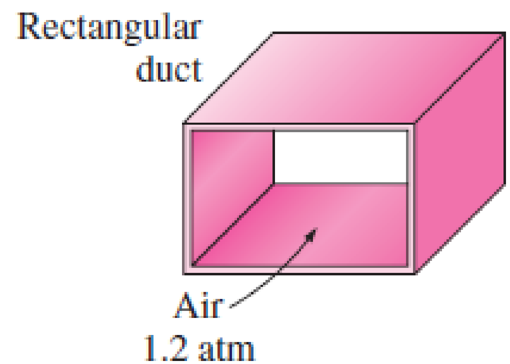
- Calculate the major and minor losses associated with pipe flow in piping networks and determine the pumping power requirements.

Examples

- Distribution of water
- Blood flow through arteries and veins
- Oil and natural gas pipelines
- Heating and cooling systems of building

- In general, flow sections of circular cross section are referred to as pipes (especially when the fluid is liquid)
- flow sections of non circular cross section are referred to as ducts (especially when the fluid is gas).
- Smaller diameter pipes are usually referred as tubes.





Circular pipes can withstand large pressure differences between the inside and the outside without undergoing any distortion, but noncircular pipes cannot.

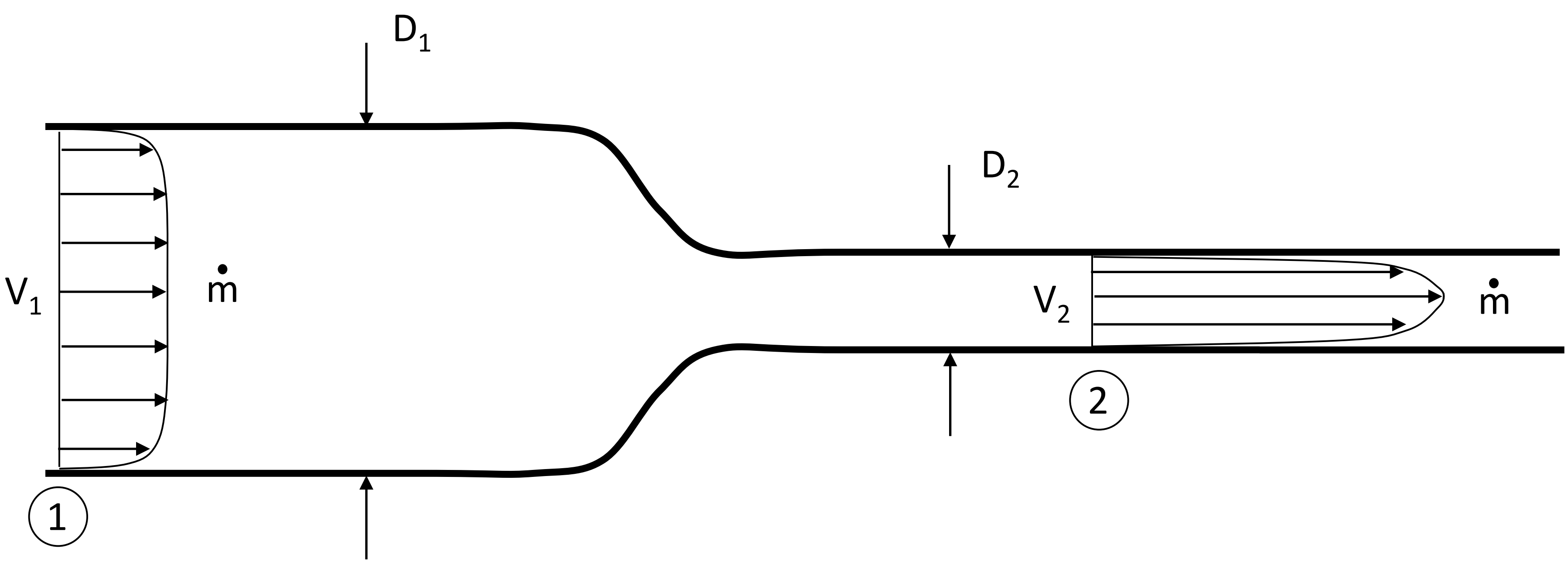
Introduction

- For pipes of constant diameter and incompressible flow
  - V_{avg} stays the same down the pipe, even if the velocity profile changes
  - Conservation of Mass

$$\dot{m} = \rho V_{avg} A = constant$$

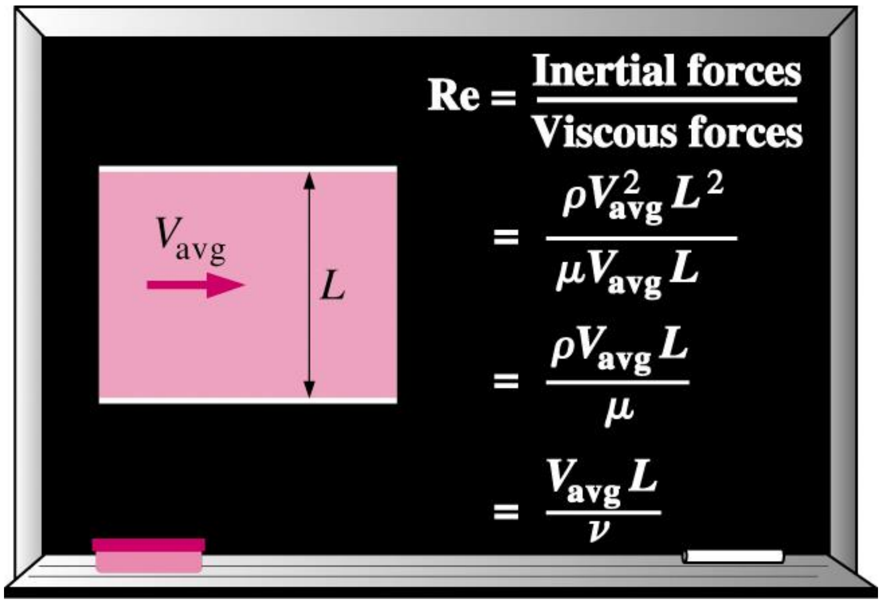
Introduction

- For pipes with variable diameter, $\dot{m}$ is still the same due to conservation of mass, but $V_1 \neq V_2$



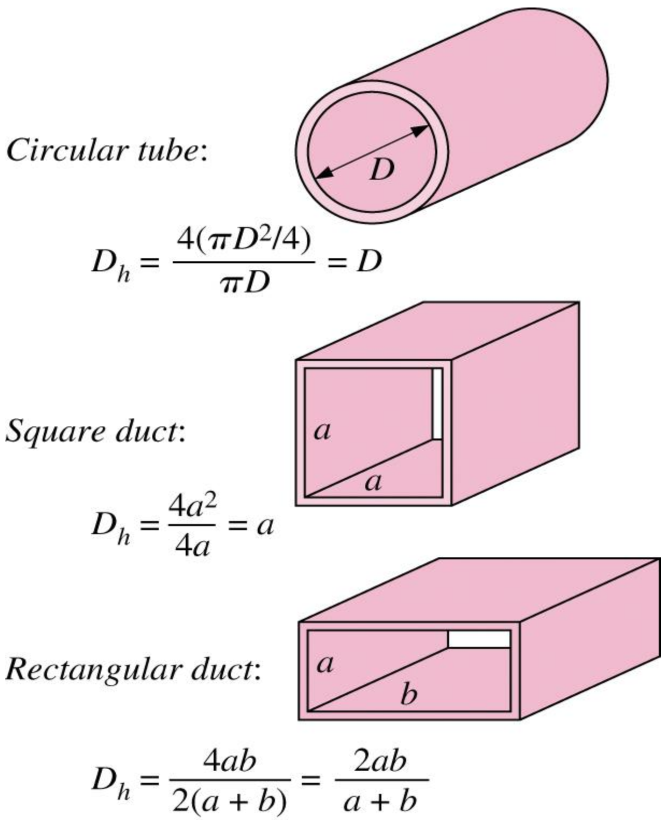
Laminar and Turbulent Flows

Definition of Reynolds number



- Critical Reynolds number (Re_{cr}) for flow in a round pipe
  - $Re < 2300 \Rightarrow$ laminar
  - $2300 \leq Re \leq 4000 \Rightarrow$ transitional
  - $Re > 4000 \Rightarrow$ turbulent
- Note that these values are approximate.
- For a given application, Re_{cr} depends upon
  - Pipe roughness
  - Vibrations
  - Upstream fluctuations, disturbances (valves, elbows, etc. that may disturb the flow)

Laminar and Turbulent Flows

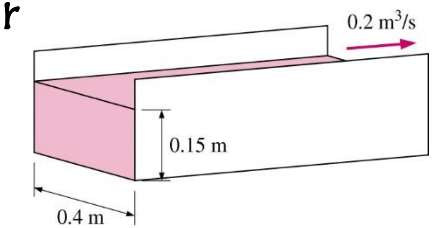


What does it mean? This channel flow is equivalent to a round pipe of diameter 0.34m (approximately).

- For non-round pipes, define the hydraulic diameter

  $D_h = 4A_c/P$

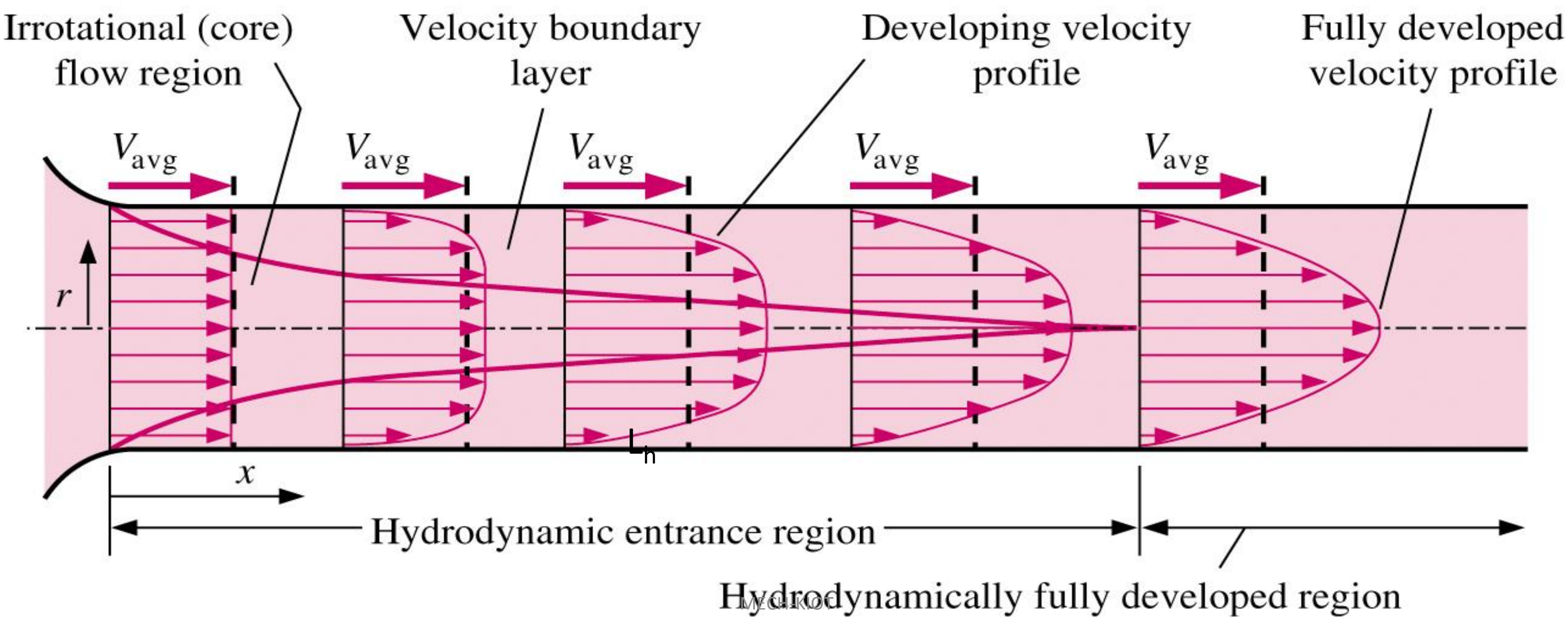
  - A_c = cross-section area
  - P = wetted perimeter



- Example: open channel
  - $Ac = 0.15 * 0.4 = 0.06m^2$
  - $P = 0.15 + 0.15 + 0.4 = 0.7m$
  - Don't count free surface, since it does not contribute to friction along pipe walls!
  - $D_h = 4A_c/P = 4*0.06/0.7 = 0.34m$

The Entrance Region

- Consider a round pipe of diameter D. The flow can be laminar or turbulent. In either case, the profile develops downstream over several diameters called the entry length L_h. L_h/D is a function of Re.



Hydrodynamic Entry Length

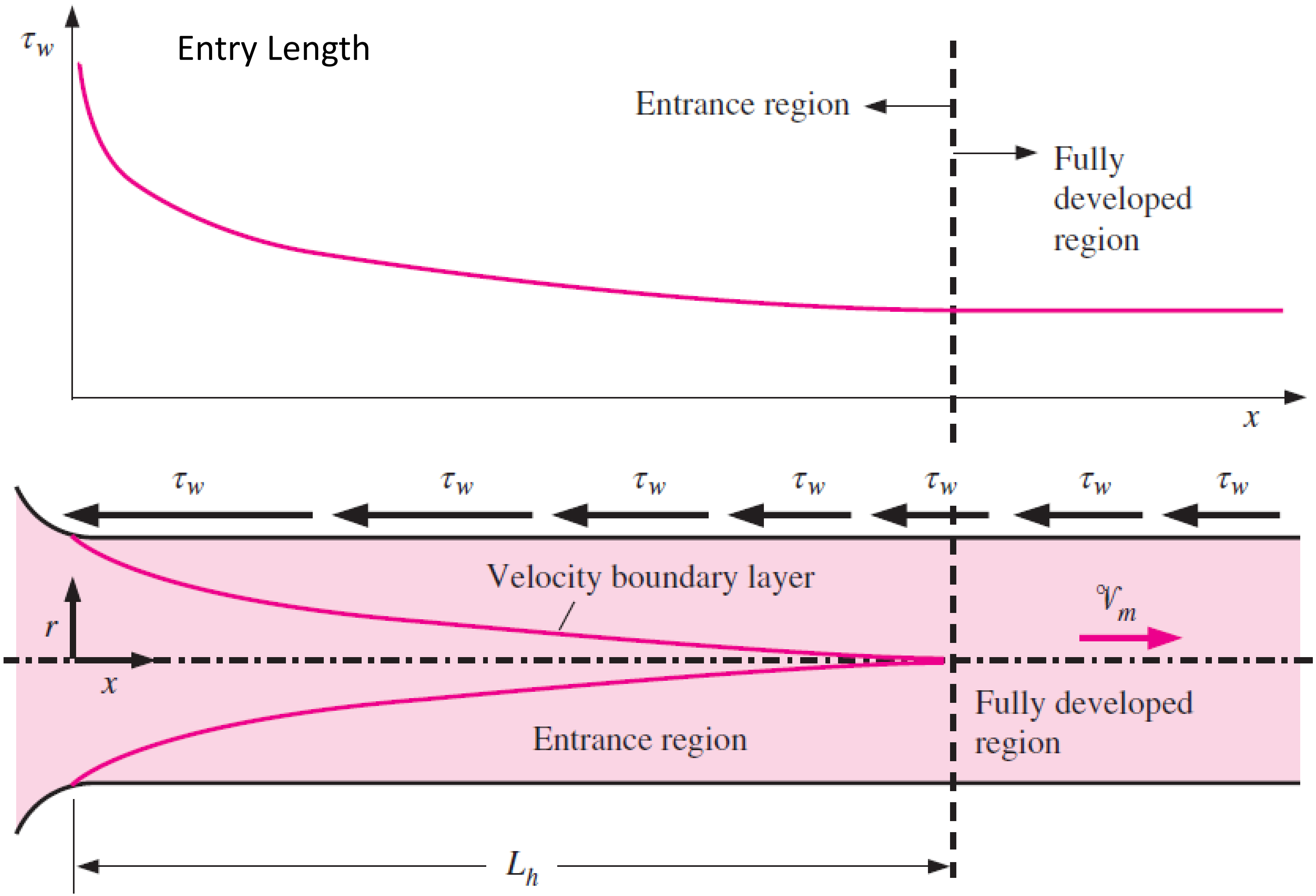
- The hydrodynamic entry length is the distance from the pipe entrance to a point where the wall shear stress (and thus the friction factor) reaches within about 2% of fully developed value.

  $L_{h,laminar} = 0.05 D Re_D$

  $L_{h,turbulent} = 1.359 D Re_D^{.25}$

  Beyond a pipe length of 10D; $L_{h,turbulent} = 10D$

Entry Length



The variation of wall shear stress in the flow direction for flow in a pipe from the entrance region into the fully developed region.

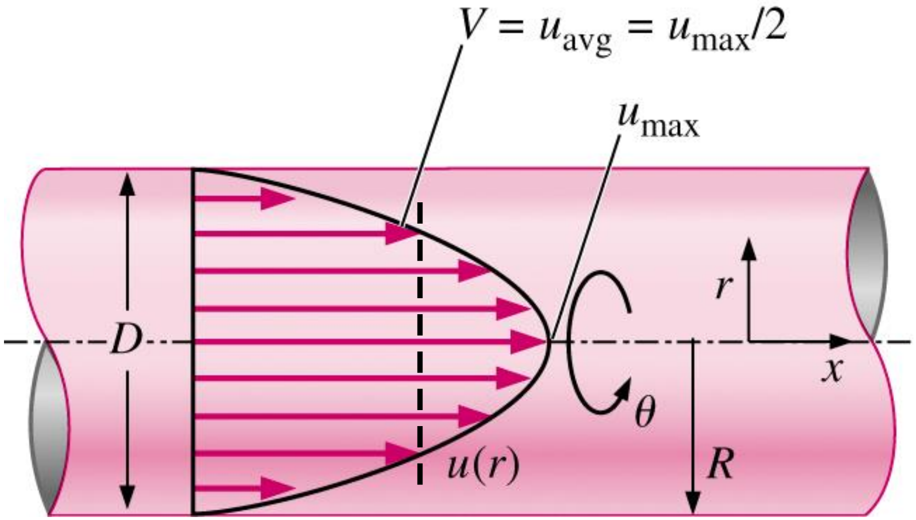
Fully Developed Pipe Flow

Comparison of laminar and turbulent flow

- There are some major differences between laminar and turbulent fully developed pipe flows

Laminar

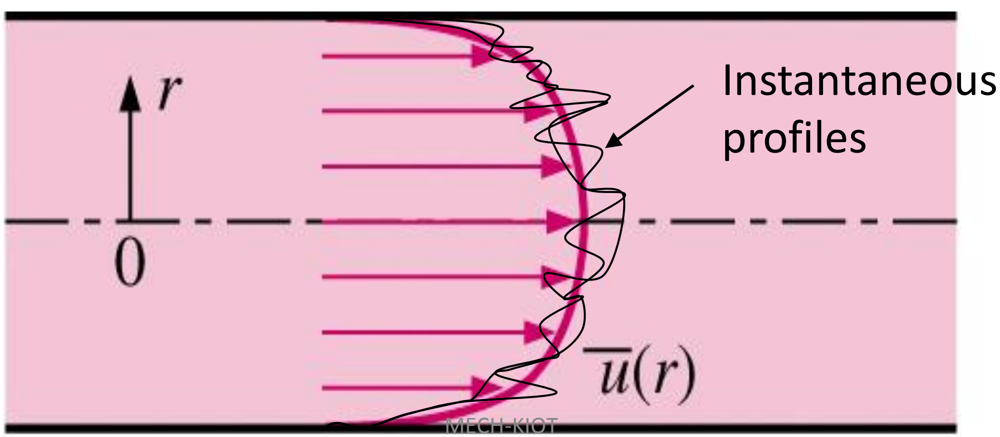
- Can solve exactly
- Flow is steady
- Velocity profile is parabolic
- Pipe roughness not important
- It turns out that $V_{avg} = 1/2U_{max}$ and $u(r) = 2V_{avg}(1 - r^2/R^2)$



Fully Developed Pipe Flow

Turbulent

- Cannot solve exactly (too complex) Flow is unsteady (3D swirling eddies), but it is steady in the mean
- Mean velocity profile is fuller (shape more like a top-hat profile, with very sharp slope at the wall)
- Pipe roughness is very important V_{avg} 85% of Umax (depends on Re)
- No analytical solution, but there are some good semi-empirical expressions that approximate the velocity profile shape.



Fully Developed Pipe Flow
Friction Factor

$$h_L = \frac{4\tau_w}{\rho g}\frac{L}{D} \qquad f = \frac{8\tau_w}{\rho V^2} \rightarrow \tau_w = f\rho V^2/8$$

$$h_L = f\frac{L}{D}\frac{V^2}{2g}$$

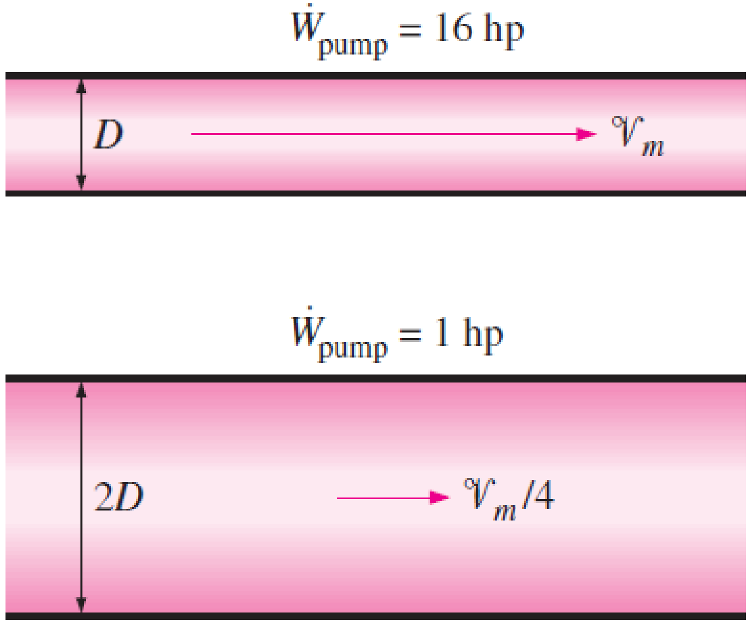
- Our problem is now reduced to solving for Darcy friction factor f
  - Recall $f = func(Re, \epsilon/D)$
    But for laminar flow, roughness does not affect the flow unless it is huge
  - Therefore
    - Laminar flow: f = 64/Re (exact)
    - Turbulent flow: Use charts or empirical equations (Moody Chart, a famous plot of f vs. Re and ε/D)

Head Loss

- In the analysis of piping system pressure losses are commonly expressed in terms of the equivalent fluid column height called head loss h_L.

- It also represents the additional height that the fluid needs to be raised by a pump inorder to overcome the frictional losses in the pipe

$$h_L = \frac{\Delta P_L}{\rho g} = \frac{f L V_{avg}^2}{2gd}$$



The pumping power requirement for a laminar flow piping system can be reduced by a factor of 16 by doubling the pipe diameter.

Once the pressure loss (or head loss) is available, the required pumping power *to overcome the pressure loss* is determined from

$$\dot{W}_{\text{pump}, L} = \dot{V}\Delta P_L = \dot{V}\rho g h_L = \dot{m} g h_L$$

where $\dot{V}$ is the volume flow rate and $\dot{m}$ is the mass flow rate.

The mean velocity for laminar flow in a horizontal pipe is,

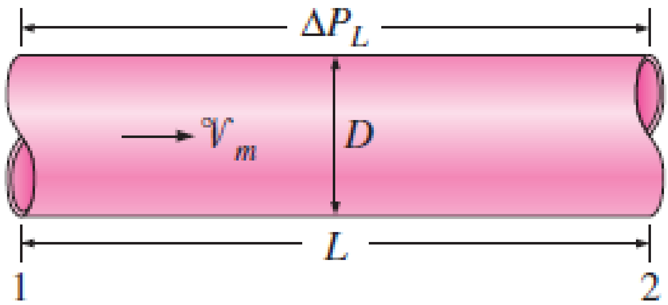
Horizontal pipe: $$\mathcal{V}_m = \frac{(P_1 - P_2)R^2}{8\mu L} = \frac{(P_1 - P_2)D^2}{32\mu L} = \frac{\Delta P D^2}{32\mu L}$$

Hagen – Poiseuille's Law

Then the volume flow rate for laminar flow through a horizontal pipe of diameter D and length L becomes

$$\textit{Horizontal pipe:} \quad \dot{V} = \mathcal{V}_m A_c = \frac{(P_1 - P_2)R^2}{8\mu L}\pi R^2 = \frac{(P_1 - P_2)\pi D^4}{128\mu L} = \frac{\Delta P \pi D^4}{128\mu L}$$

This equation is known as **Poiseuille's Law,** and this flow is called *Hagen-Poiseuille flow* in honor of the works of G. Hagen (1797–1839) and J. Poiseuille (1799–1869) on the subject. Note from Eq. that *for a specified flow rate, the pressure drop and thus the required pumping power is proportional to the length of the pipe and the viscosity of the fluid, but it is inversely proportional to the fourth power of the radius (or diameter) of the pipe.*



Pressure loss: $\Delta P_L = f \frac{L}{D} \frac{\rho \mathcal{V}_m^2}{2}$

Head loss: $h_L = \frac{\Delta P_L}{\rho g} = f \frac{L}{D} \frac{\mathcal{V}_m^2}{2g}$

The relation for pressure loss (and head loss) is one of the most general relations in fluid mechanics, and it is valid for laminar or turbulent flows, circular or noncircular pipes, and smooth or rough surfaces.

Horizontal pipe: $\dot{V} = \frac{\Delta P \pi D^4}{128 \mu L}$

Inclined pipe: $\dot{V} = \frac{(\Delta P - \rho g L \sin\theta) \pi D^4}{128 \mu L}$

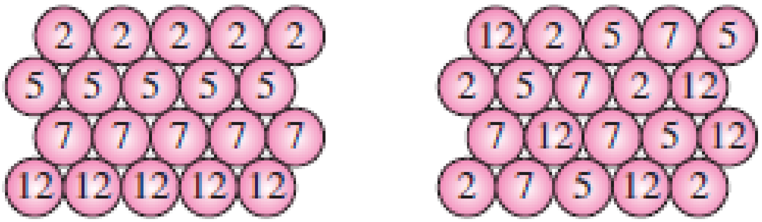
Uphill flow: $\theta > 0$ and $\sin\theta > 0$

Downhill flow: $\theta < 0$ and $\sin\theta < 0$

The relations developed for fully developed laminar flow through horizontal pipes can also be used for inclined pipes by replacing ΔP with $\Delta P - \rho g L \sin\theta$.

Turbulent flows in pipes
Eddies

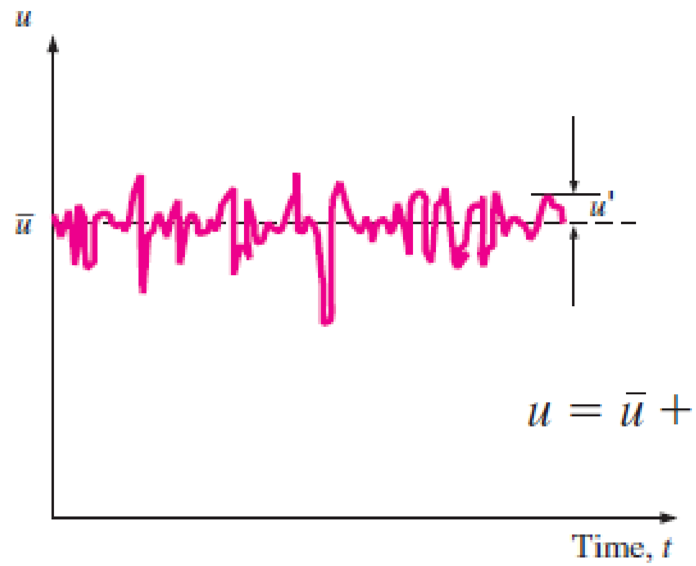
Turbulent flow is characterized by random and rapid fluctuations of swirling fluid particles, called *eddies*, throughout the flow.



(*a*) Before turbulence

(*b*) After turbulence

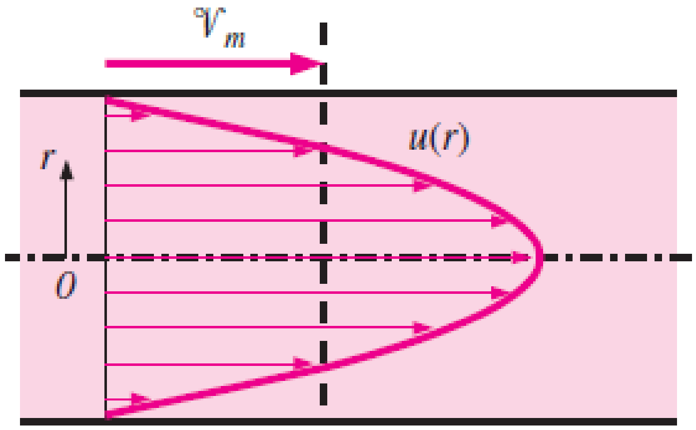
The intense mixing in turbulent flow brings fluid particles at different momentums into close contact, and thus enhances momentum transfer.



$$u = \bar{u} + u'$$

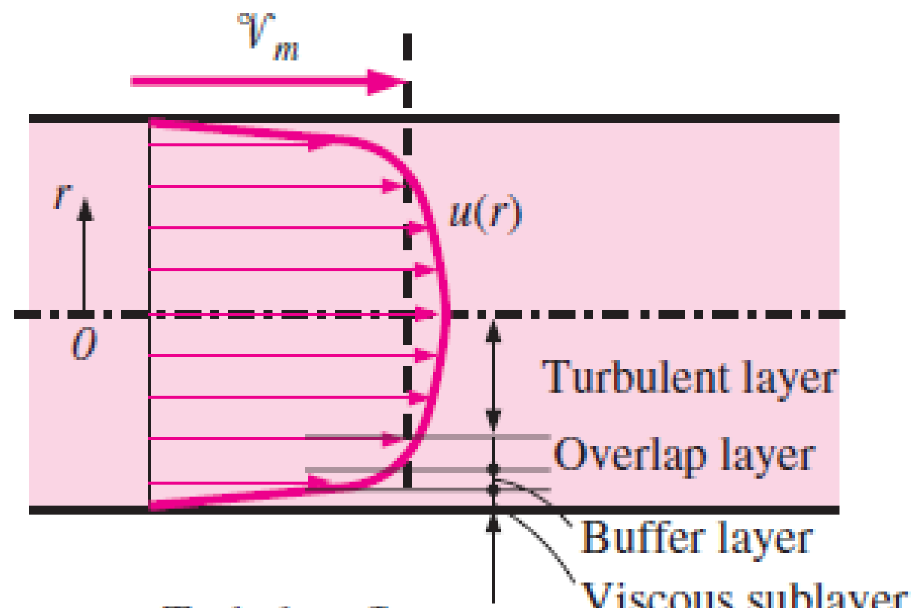
Fluctuations of the velocity component u with time at a specified location in turbulent flow.

Turbulent Velocity Profile



Laminar flow

the velocity profile is parabolic in laminar flow



Turbulent flow

fuller in turbulent flow, with a sharp drop near the pipe wall

Turbulent Velocity Profile

viscous effects are dominant is the **viscous** (or **laminar** or **linear** or **wall**) sublayer. The velocity profile in this layer is very nearly *linear*, and the flow is streamlined.

Next to the viscous sublayer is the **buffer layer,** in which turbulent effects are becoming significant, but the flow is still dominated by viscous effects.

Above the buffer layer is the **overlap** (or **transition**) **layer,** also called the **inertial sublayer,** in which the turbulent effects are much more significant, but still not dominant.

Above that is the **outer** (or **turbulent**) **layer** in the remaining part of the flow in which turbulent effects dominate over molecular diffusion (viscous) effects.

Fully Developed Pipe Flow - Friction Factor

- Moody chart was developed for circular pipes, but can be used for non-circular pipes using hydraulic diameter

- Colebrook equation is a curve-fit of the data which is convenient for computations

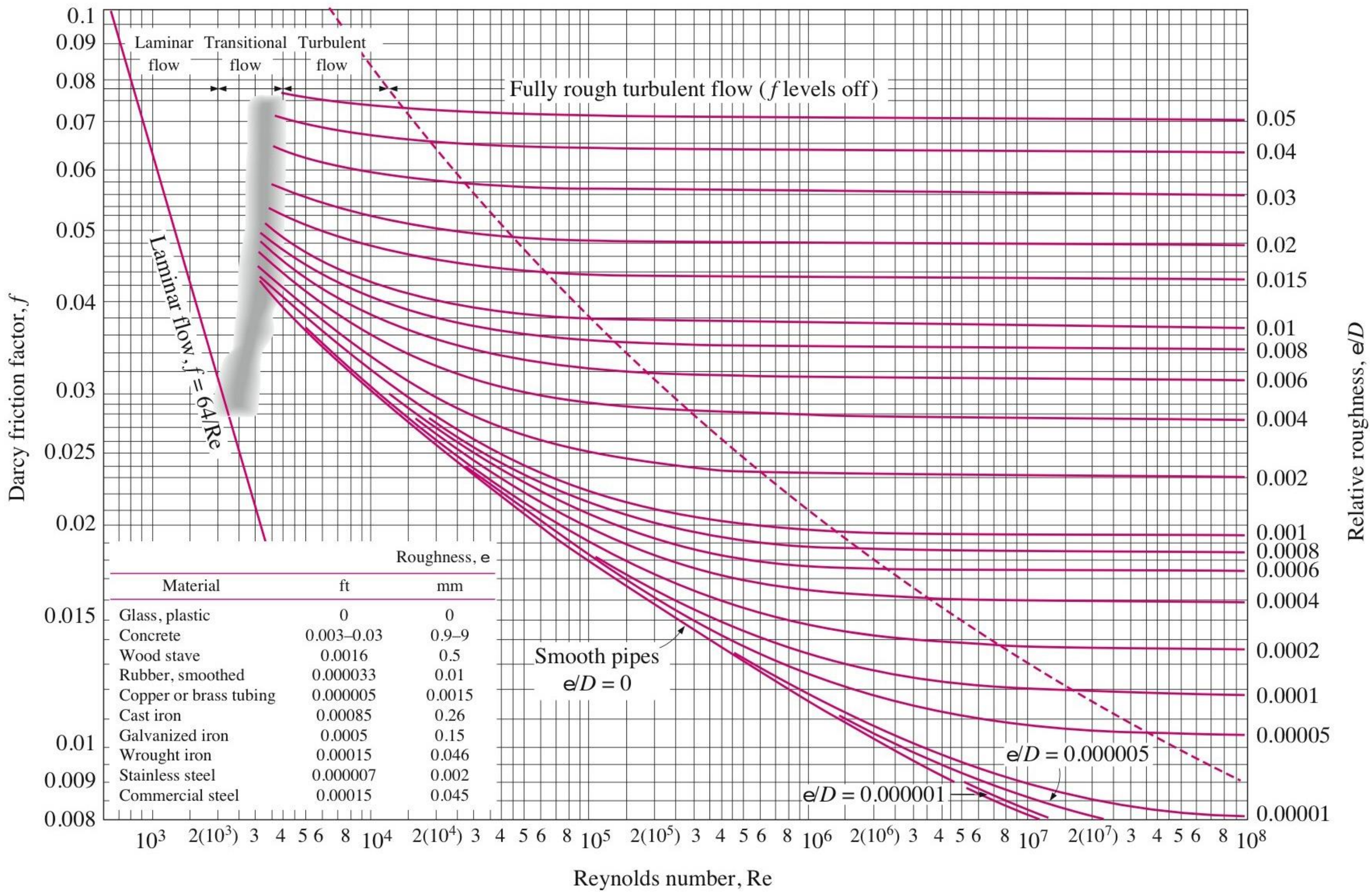
$$\frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}} \right)$$

- Both Moody chart and Colebrook equation are accurate to ±15% due to roughness size, experimental error, curve fitting of data, etc.

- S.E. Haaland equation which are 2% error of the Colebrook eqaution.

$$\frac{1}{\sqrt{f}} \approx -1.8 \log \left[\frac{6.9}{\text{Re}} + \left(\frac{\varepsilon/D}{3.7} \right)^{1.11} \right]$$

The Moody Chart



Material	Roughness, e ft	Roughness, e mm
Glass, plastic	0	0
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

Equivalent roughness values for new commercial pipes*

Material	Roughness, ε ft	Roughness, ε mm
Glass, plastic	0 (smooth)	
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

*The uncertainty in these values can be as much as ±60 percent.

Relative Roughness, ε/D	Friction Factor, f
0.0*	0.0119
0.00001	0.0119
0.0001	0.0134
0.0005	0.0172
0.001	0.0199
0.005	0.0305
0.01	0.0380
0.05	0.0716

*Smooth surface. All values are for Re = 10^6, and are calculated from Colebrook equation.

The friction factor is minimum for a smooth pipe and increases with roughness.

Types of Fluid Flow Problems

- Explicit relations have been developed which eliminate iteration. They are useful for quick, direct calculation, but introduce an additional 2% error
- Swamee and Jain Relation

$$h_L = 1.07\frac{\dot{\mathcal{V}}^2 L}{gD^5}\left\{\ln\left[\frac{\epsilon}{3.7D} + 4.62\left(\frac{\nu D}{\dot{\mathcal{V}}}\right)^{0.9}\right]\right\}^{-2} \qquad \begin{array}{c} 10^{-6} < \epsilon/D < 10^{-2} \\ 3000 < Re < 3\times10^{8} \end{array}$$

$$\dot{\mathcal{V}} = -0.965\left(\frac{gD^5 h_L}{L}\right)^{0.5}\ln\left[\frac{\epsilon}{3.7D} + \left(\frac{3.17\nu^2 L}{gD^3 h_L}\right)^{0.5}\right] \qquad Re > 2000$$

$$D = 0.66\left[\epsilon^{1.25}\left(\frac{L\dot{\mathcal{V}}^2}{gh_L}\right)^{4.75} + \nu\dot{\mathcal{V}}^{9.4}\left(\frac{L}{gh_L}\right)^{5.2}\right]^{0.04} \qquad \begin{array}{c} 10^{-6} < \epsilon/D < 10^{-2} \\ 5000 < Re < 3\times10^{8} \end{array}$$

Types of Fluid Flow Problems

1. Determining the **pressure drop** (or head loss) when the pipe length and diameter are given for a specified flow rate (or velocity).

2. Determining the **flow rate** when the pipe length and diameter are given for a specified pressure drop (or head loss).

3. Determining the **pipe diameter** when the pipe length and flow rate are given for a specified pressure drop (or head loss).

Problem type	Given	Find
1	L, D, $\dot{V}$	ΔP (or h_L)
2	L, D, ΔP	$\dot{V}$
3	L, ΔP, $\dot{V}$	D

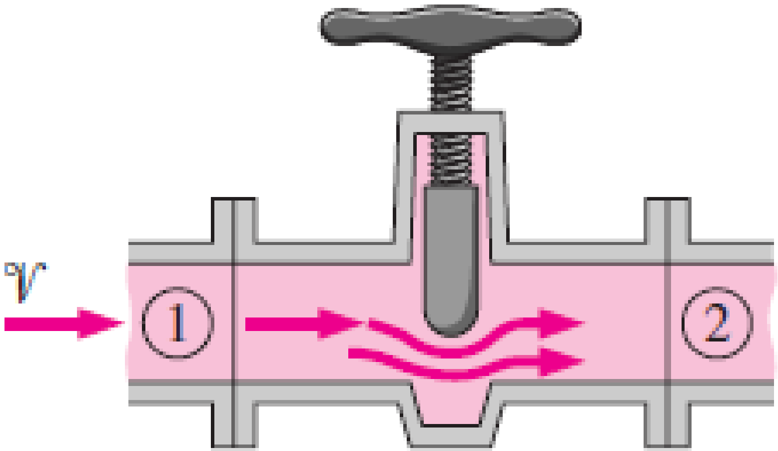
Minor Losses

- Piping systems include fittings, valves, bends, elbows, tees, inlets, exits, enlargements, and contractions.
- These components interrupt the smooth flow of fluid and cause additional losses because of flow separation and mixing.
- We introduce a relation for the minor losses associated with these components

$$h_L = K_L \frac{V^2}{2g}$$

- K_L is the loss coefficient.
- Is different for each component.
- Is assumed to be independent of Re.
- Typically provided by manufacturer or generic table

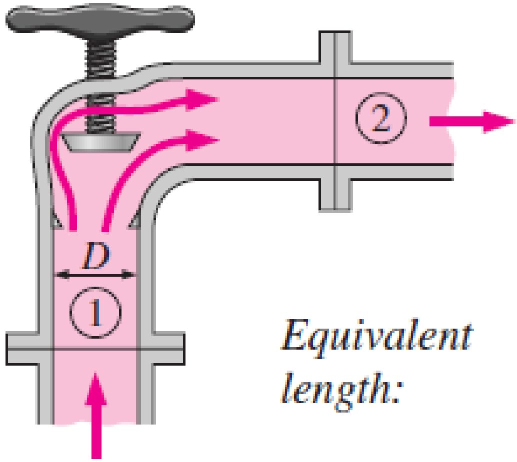
Minor losses are usually expressed in terms of the **loss coefficient** K_L,



$$\Delta P_L = \Delta P = P_1 - P_2$$

$$K_L = \frac{\Delta P_L}{\frac{1}{2}\rho \mathcal{V}^2}$$

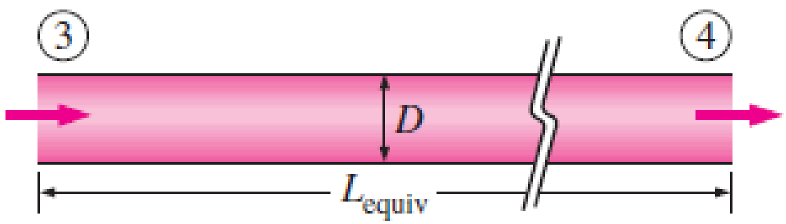
The loss coefficient of a component (such as the gate valve shown) is determined by measuring the pressure loss it causes and dividing it by the dynamic pressure in the pipe.



Equivalent length:

$$h_L = K_L \frac{\mathcal{V}^2}{2g} = f \frac{L_{\text{equiv}}}{D} \frac{\mathcal{V}^2}{2g} \quad \rightarrow \quad L_{\text{equiv}} = \frac{D}{f} K_L$$

$$\Delta P = P_1 - P_2 = P_3 - P_4$$



The head loss caused by a component (such as the angle valve shown) is equivalent to the head loss caused by a section of the pipe whose length is the equivalent length.

Minor Losses

- Total head loss in a system is comprised of major losses (in the pipe sections) and the minor losses (in the components)

$$h_L = h_{L,major} + h_{L,minor}$$

$$h_L = \underbrace{\sum_i f_i \frac{L_i}{D_i} \frac{V_i^2}{2g}}_{\text{i pipe sections}} + \underbrace{\sum_j K_{L,j} \frac{V_j^2}{2g}}_{\text{j components}}$$

- If the pipin

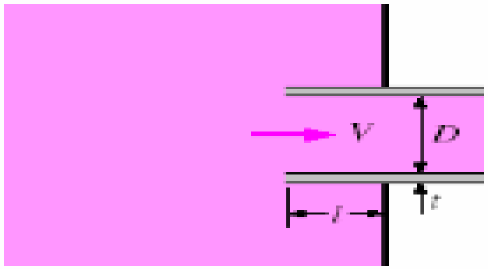
$$h_L = \left(f \frac{L}{D} + \sum K_L \right) \frac{V^2}{2g}$$

Minor Losses

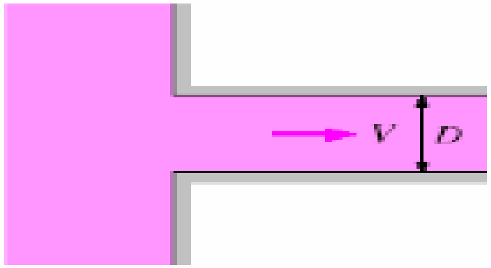
Here are some sample loss coefficients for various minor loss components. More values are listed in Table 8-4, page 350 of the Çengel-Cimbala textbook:

Pipe Inlet

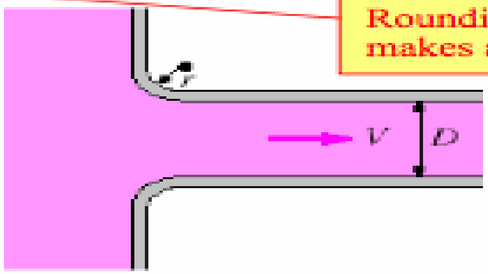
Reentrant: $K_L = 0.80$
($t \ll D$ and $l \approx 0.1D$)



Sharp-edged: $K_L = 0.50$



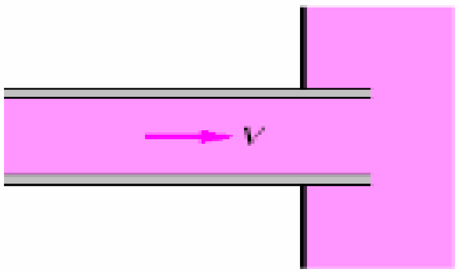
Well-rounded ($r/D > 0.2$): $K_L = 0.03$
Slightly rounded ($r/D = 0.1$): $K_L = 0.12$
(see Fig. 8–36)



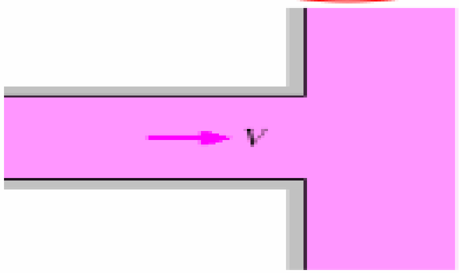
Rounding of an inlet makes a big difference.

Pipe Exit

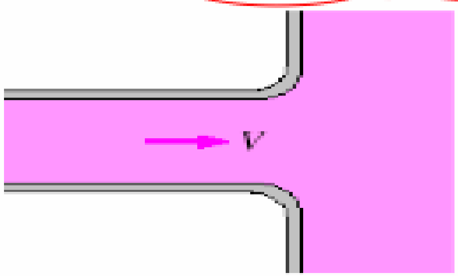
Reentrant: $K_L = \alpha$



Sharp-edged: $K_L = \alpha$



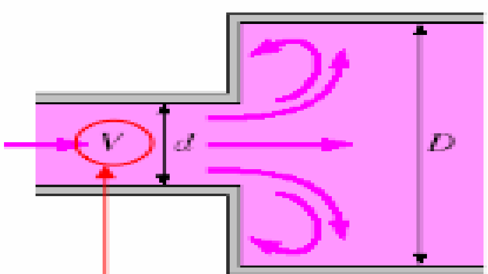
Rounded: $K_L = \alpha$



Rounding of an outlet makes no difference.

Sudden Expansion and Contraction (based on the velocity in the smaller-diameter pipe)

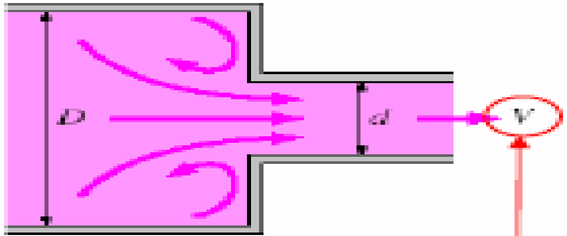
Sudden expansion: $K_L = \left(1 - \frac{d^2}{D^2}\right)^2$

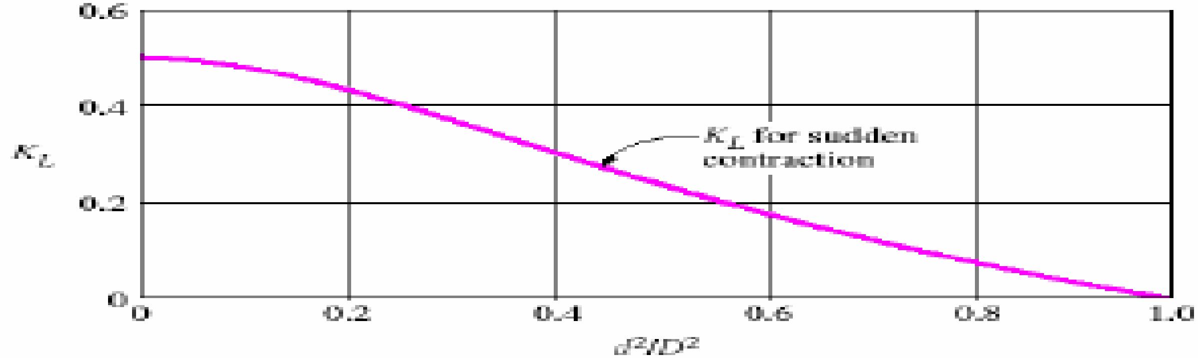


Note that the *larger velocity* (the velocity associated with the *smaller pipe section*) is used by convention in the equation for minor head loss, i.e.,

$$h_{L,\ minor} = K_L \frac{V^2}{2g}.$$

Sudden contraction: See chart.





Note: These are backwards. The K_L values listed for Expansion should be those for Contraction, and vice-versa.

Note again that the *larger velocity* (the velocity associated with the *smaller pipe section*) is used by convention in the equation for minor head loss, i.e., $h_{L,\text{minor}} = K_L \dfrac{V^2}{2g}$.

Gradual Expansion and Contraction (based on the velocity in the smaller-diameter pipe)

Expansion:

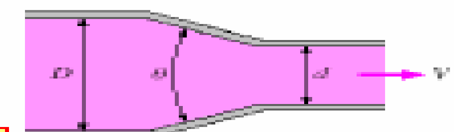
$K_L = 0.02$ for $\theta = 20°$
$K_L = 0.04$ for $\theta = 45°$
$K_L = 0.07$ for $\theta = 60°$

These are for contractions

Contraction (for $\theta = 20°$):

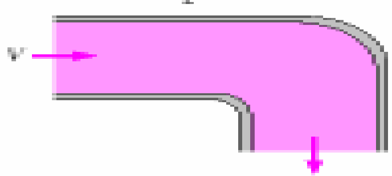
$K_L = 0.30$ for $d/D = 0.2$
$K_L = 0.25$ for $d/D = 0.4$
$K_L = 0.15$ for $d/D = 0.6$
$K_L = 0.10$ for $d/D = 0.8$

These are for expansions

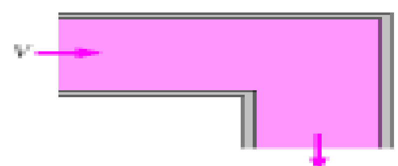


Bends and Branches

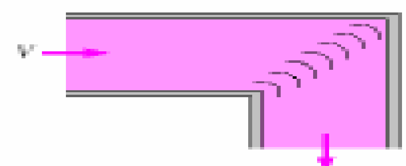
90° smooth bend:
Flanged: $K_L = 0.3$
Threaded: $K_L = 0.9$



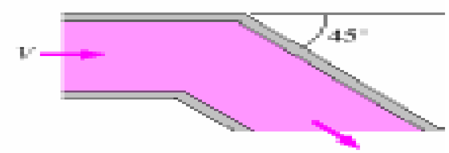
90° miter bend (without vanes): $K_L = 1.1$



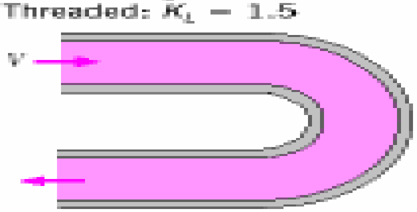
90° miter bend (with vanes): $K_L = 0.2$



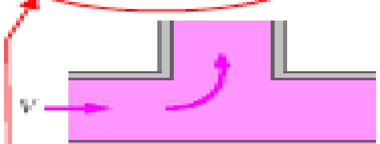
45° threaded elbow:
$K_L = 0.4$



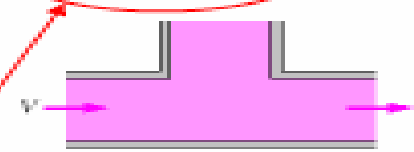
180° return bend:
Flanged: $K_L = 0.2$
Threaded: $K_L = 1.5$



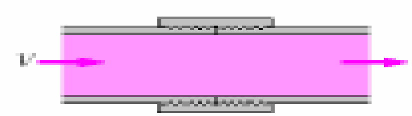
Tee (branch flow):
Flanged: $K_L = 1.0$
Threaded: $K_L = 2.0$



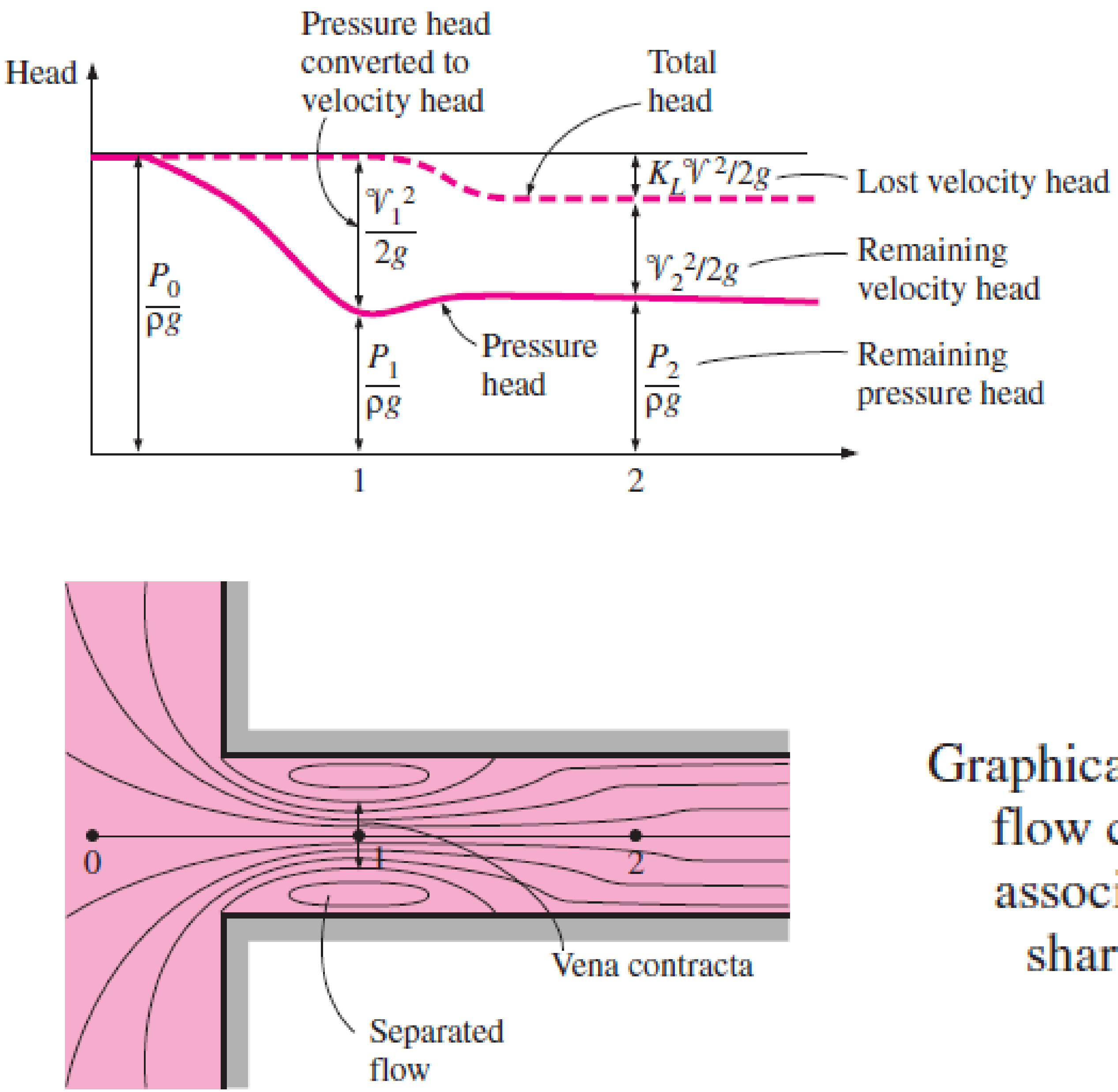
Tee (line flow):
Flanged: $K_L = 0.2$
Threaded: $K_L = 0.9$



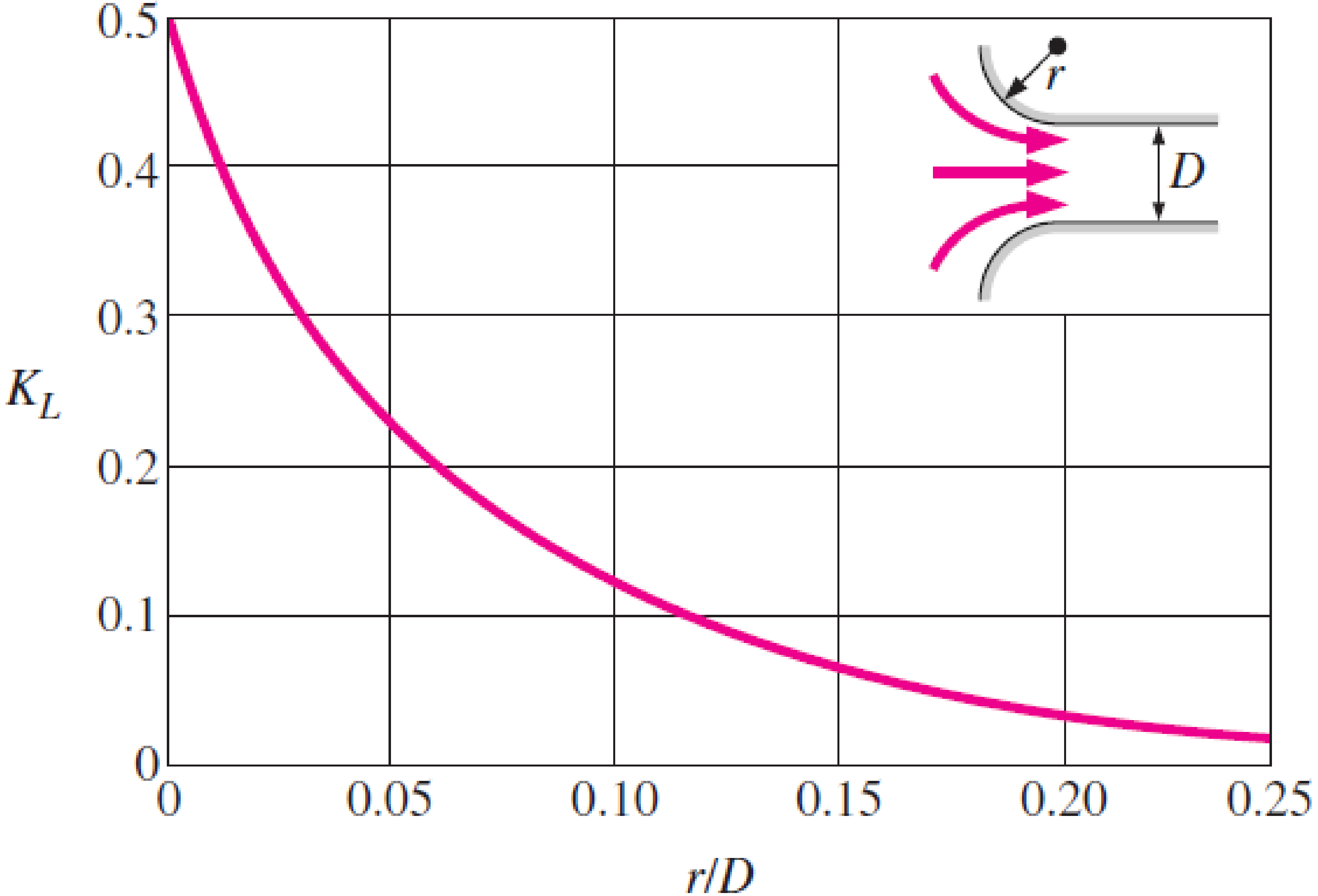
Threaded union:
$K_L = 0.08$



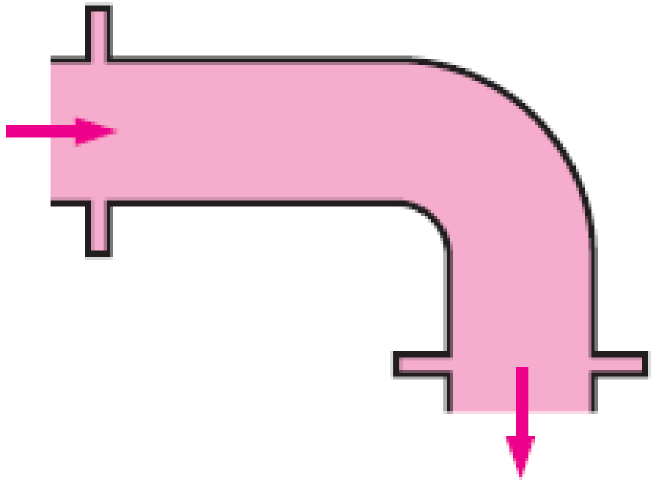
For tees, there are two values of K_L, one for *branch flow* and one for *line flow*.



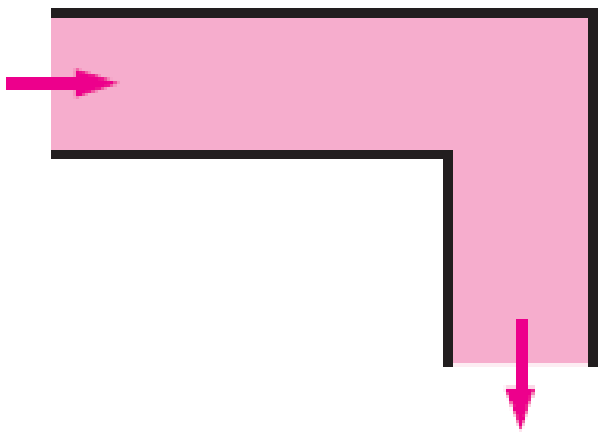
Graphical representation of flow contraction and the associated head loss at a sharp-edged pipe inlet.



The effect of rounding of a pipe inlet on the loss coefficient (from *ASHRAE Handbook of Fundamentals*).

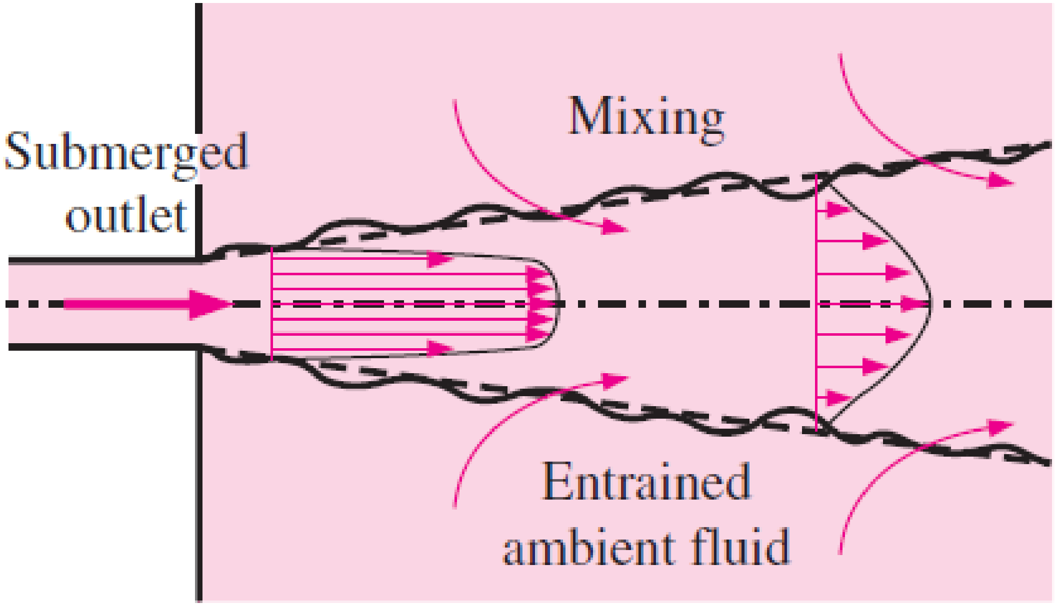


Flanged elbow
$K_L = 0.3$



Sharp turn
$K_L = 1.1$

The losses during changes of direction can be minimized by making the turn "easy" on the fluid by using circular arcs instead of sharp turns.

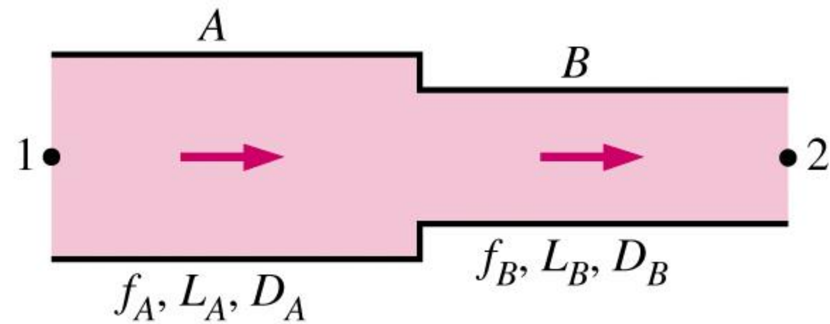


$$K_L = \left(1 - \frac{A_{small}}{A_{large}}\right)^2 \qquad \text{(sudden expansion)}$$

All of the kinetic energy of the flow is "lost" (turned into thermal energy) through friction as the jet decelerates and mixes with ambient fluid downstream of a submerged outlet.

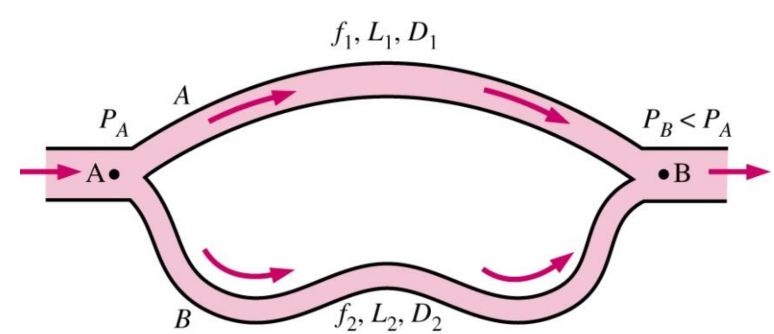
Piping Networks and Pump Selection

- Two general types of networks
  - Pipes in series
    - Volume flow rate is constant
    - Head loss is the summation of parts



$$\dot{V}_A = \dot{V}_B$$
$$h_{L,\,1\text{-}2} = h_{L,\,A} + h_{L,\,B}$$

  - Pipes in parallel
    - Volume flow rate is the sum of the components
    - Pressure loss across all branches is the same



$$h_{L,\,1} = h_{L,\,2}$$
$$\dot{V}_A = \dot{V}_1 + \dot{V}_2 = \dot{V}_B$$

Piping Networks and Pump Selection

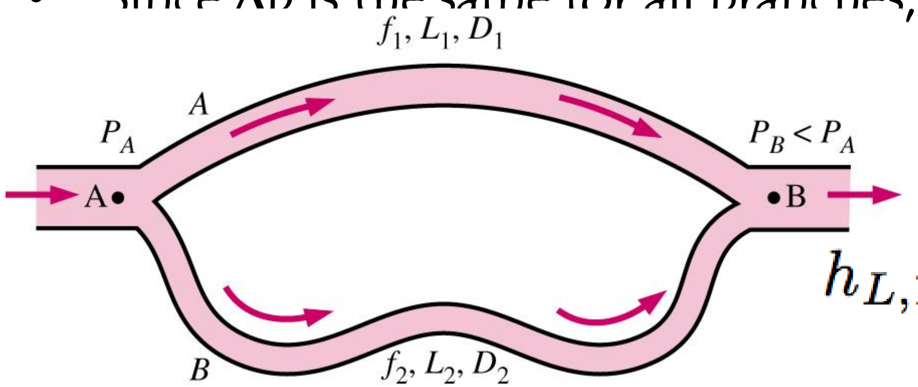
- For parallel pipes, perform CV analysis between points A and B

$$V_A = V_B$$

$$\frac{P_A}{\rho g} + \alpha_1 \frac{V_A^2}{2g} + z_A = \frac{P_B}{\rho g} + \alpha_2 \frac{V_B^2}{2g} + z_B + h_L$$

$$h_L = \frac{\Delta P}{\rho g}$$

- Since Δp is the same for all branches, head loss in all branches is the same



$$h_{L,1} = h_{L,2}$$
$$\dot{V}_A = \dot{V}_1 + \dot{V}_2 = \dot{V}_B$$

$$h_{L,1} = h_{L,2} \Rightarrow f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} = f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g}$$

Piping Networks and Pump Selection

- Head loss relationship between branches allows the following ratios to be developed

$$\frac{V_1}{V_2} = \left(\frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\right)^{\frac{1}{2}} \qquad \frac{\dot{\mathcal{V}}_1}{\dot{\mathcal{V}}_2} = \frac{D_1^2}{D_2^2}\left(\frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\right)^{\frac{1}{2}}$$

- Real pipe systems result in a system of non-linear equations. Very easy to solve with EES!
- Note: the analogy with electrical circuits should be obvious
  - Flow flow rate (VA) : current (I)
  - Pressure gradient (Δp) : electrical potential (V)
  - Head loss (hL): resistance (R), however hL is very nonlinear

Piping Networks and Pump Selection

- When a piping system involves pumps and/or turbines, pump and turbine head must be included in the energy equation

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + \boxed{h_{pump,u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + \boxed{h_{turbine,e}} + h_L$$

- The useful head of the pump (hpump,u) or the head extracted by the turbine (hturbine,e), are functions of volume flow rate, i.e., they are not constants.
- Operating point of system is where the system is in balance, e.g., where pump head is equal to the head losses.

Chapter 09

Boundary layer theory

Objectives

- To know about the development of boundary layers in external and internal flows
- Predict boundary layer thickness and other boundary layer properties.
- Demonstrate the phenomena of turbulence, separation and recirculation of flow

Boundary layer theory

- The narrow region, near the solid surface, over which velocity gradient and shear stresses are large is known as boundary layer.

- The study of velocity gradients, shear stresses, forces and energy loss in the boundary layer is called as boundary layer theory.

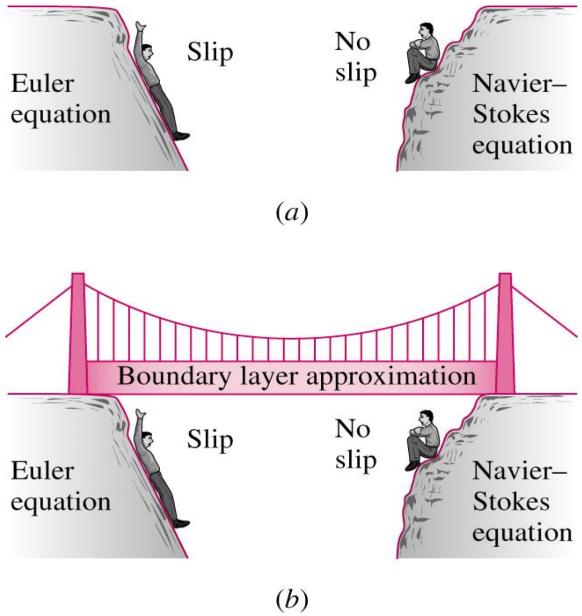
- A layer of fluid near the surface of the body in which the velocity changes from zero on the surface to the free-stream value.

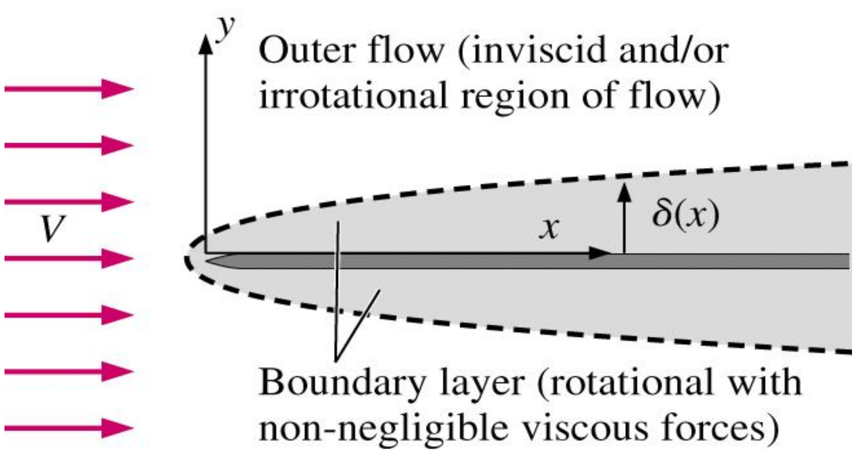
- Prandtl's insight into this phenomenon and his subsequent development of boundary layer theory are milestones in the development of fluid mechanics

- the thickness of the boundary layer increases in the downstream direction

- just downstream of the nose of the plate the boundary layer is observed to be laminar, but at some point in downstream transition occurs and the boundary layer becomes turbulent.

Boundary Layer (BL) Approximation

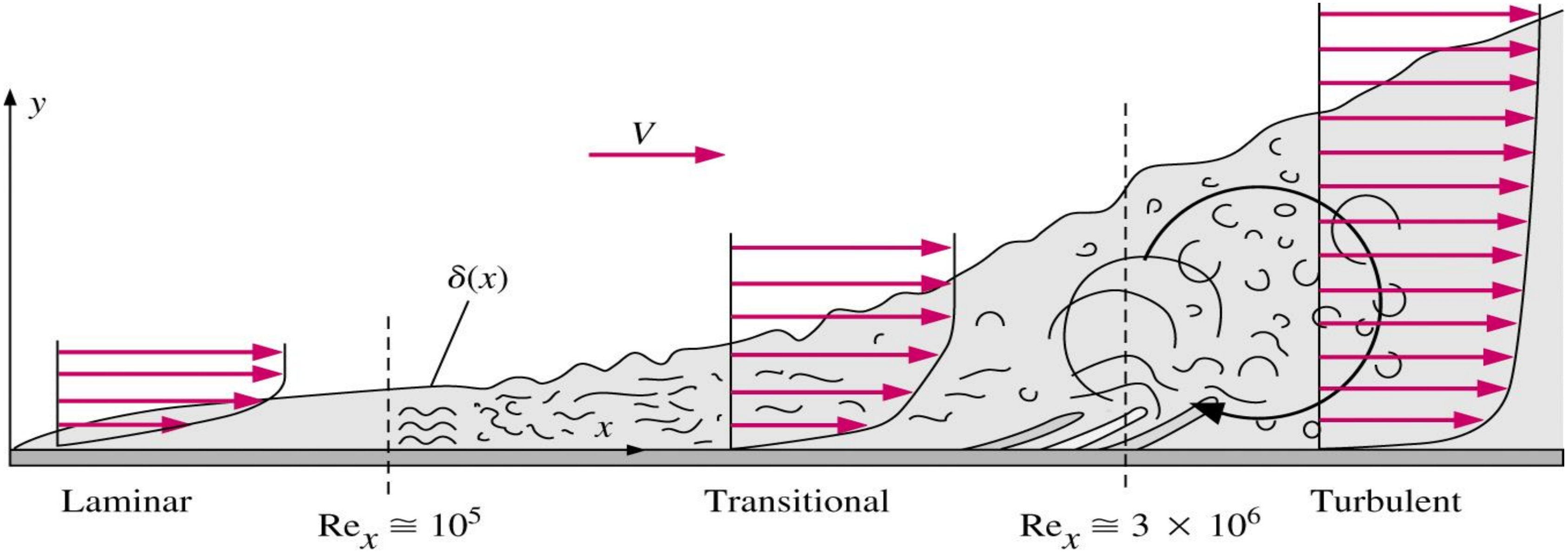


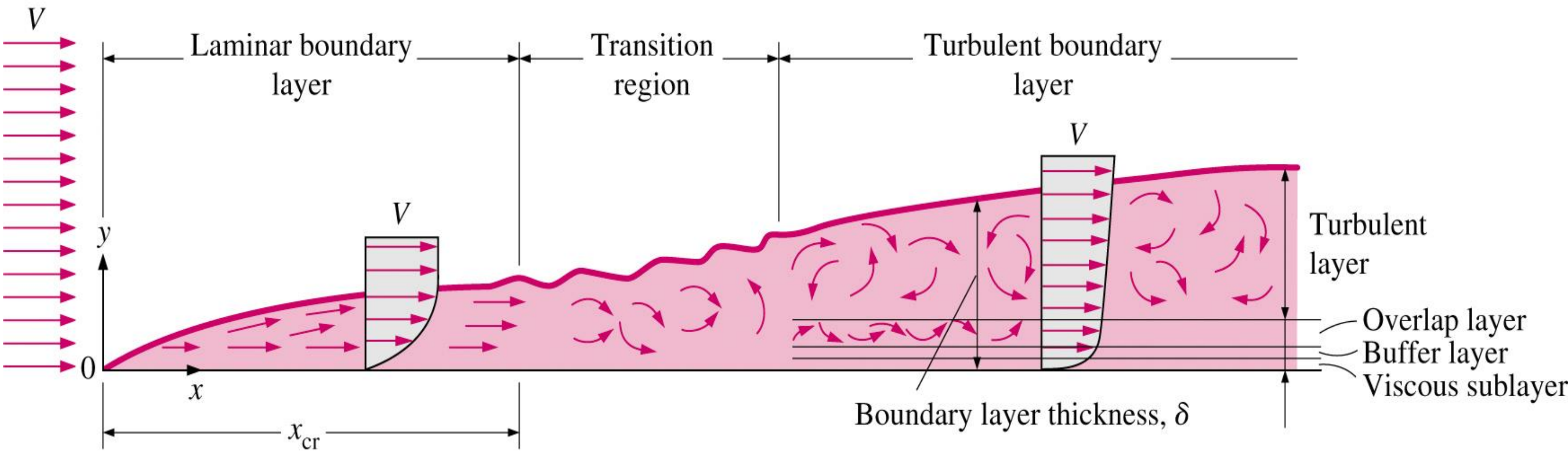


- BL approximation bridges the gap between the Euler and NS equations, and between the slip and no-slip BC at the wall.

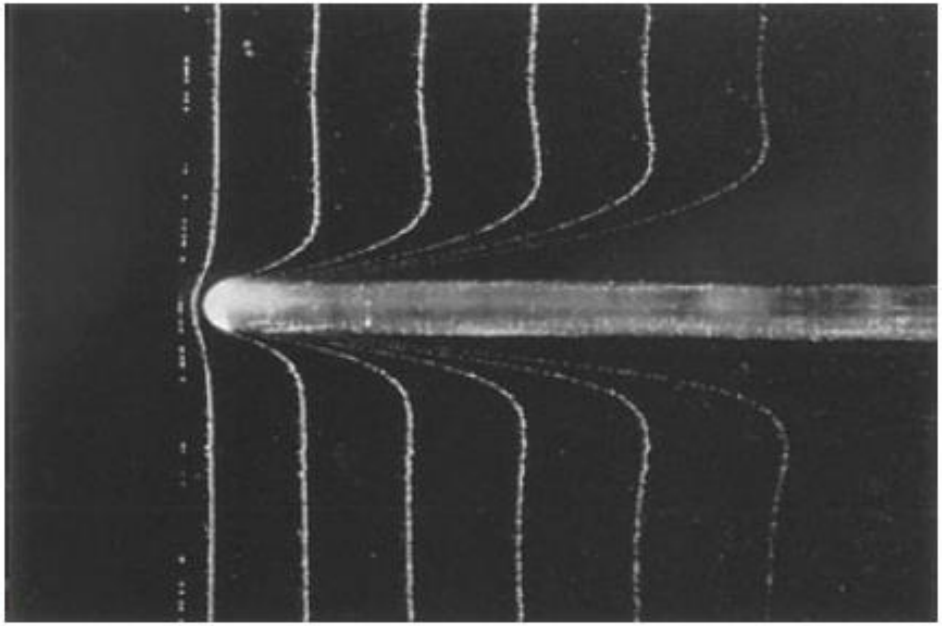
- Prandtl (1904) introduced the BL approximation

Boundary Layer Region over a Flat Plate

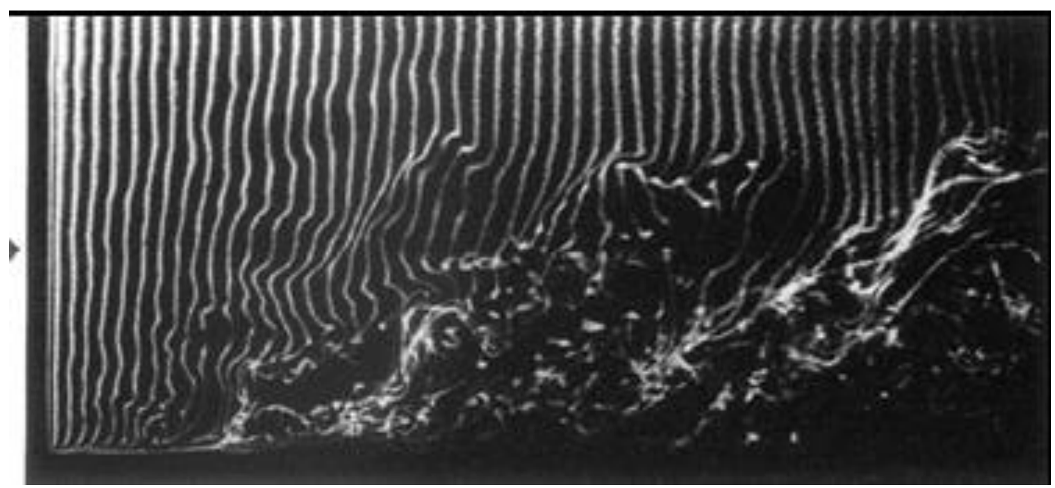




(A)

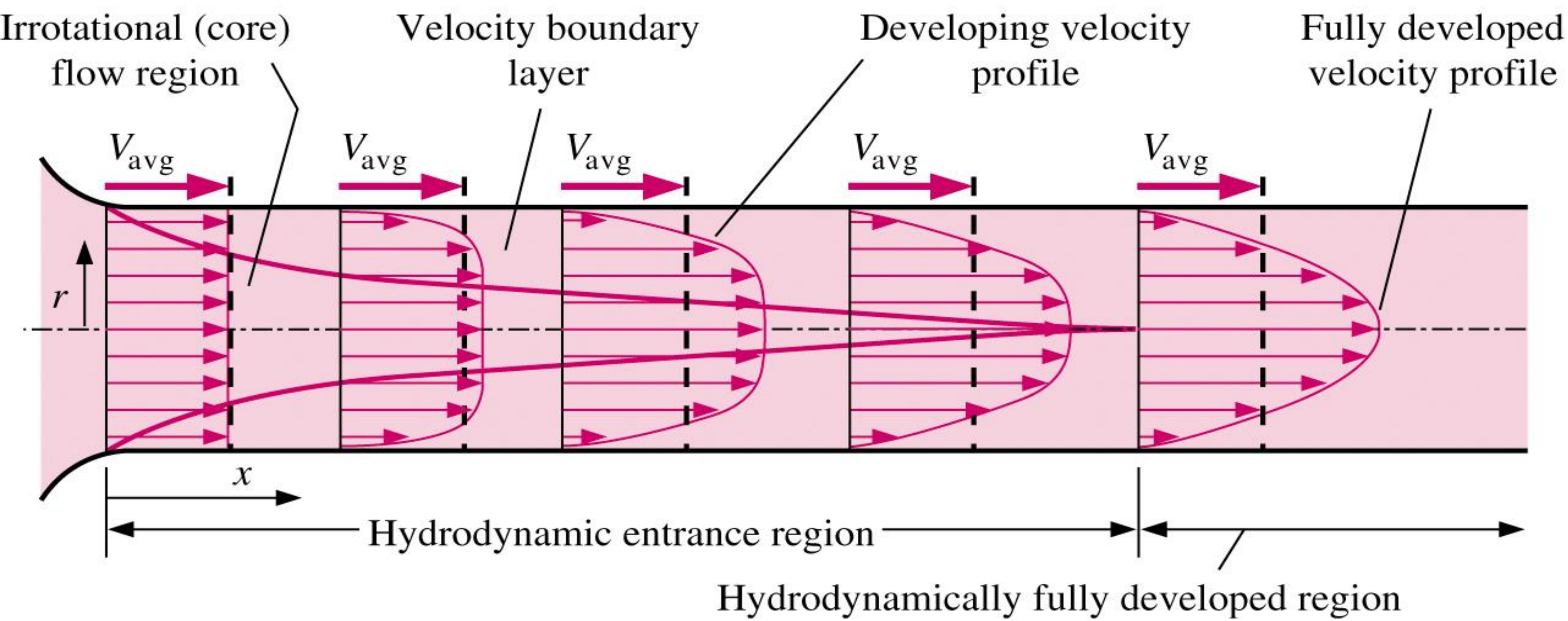


- Laminar boundary layer near the nose of the plate

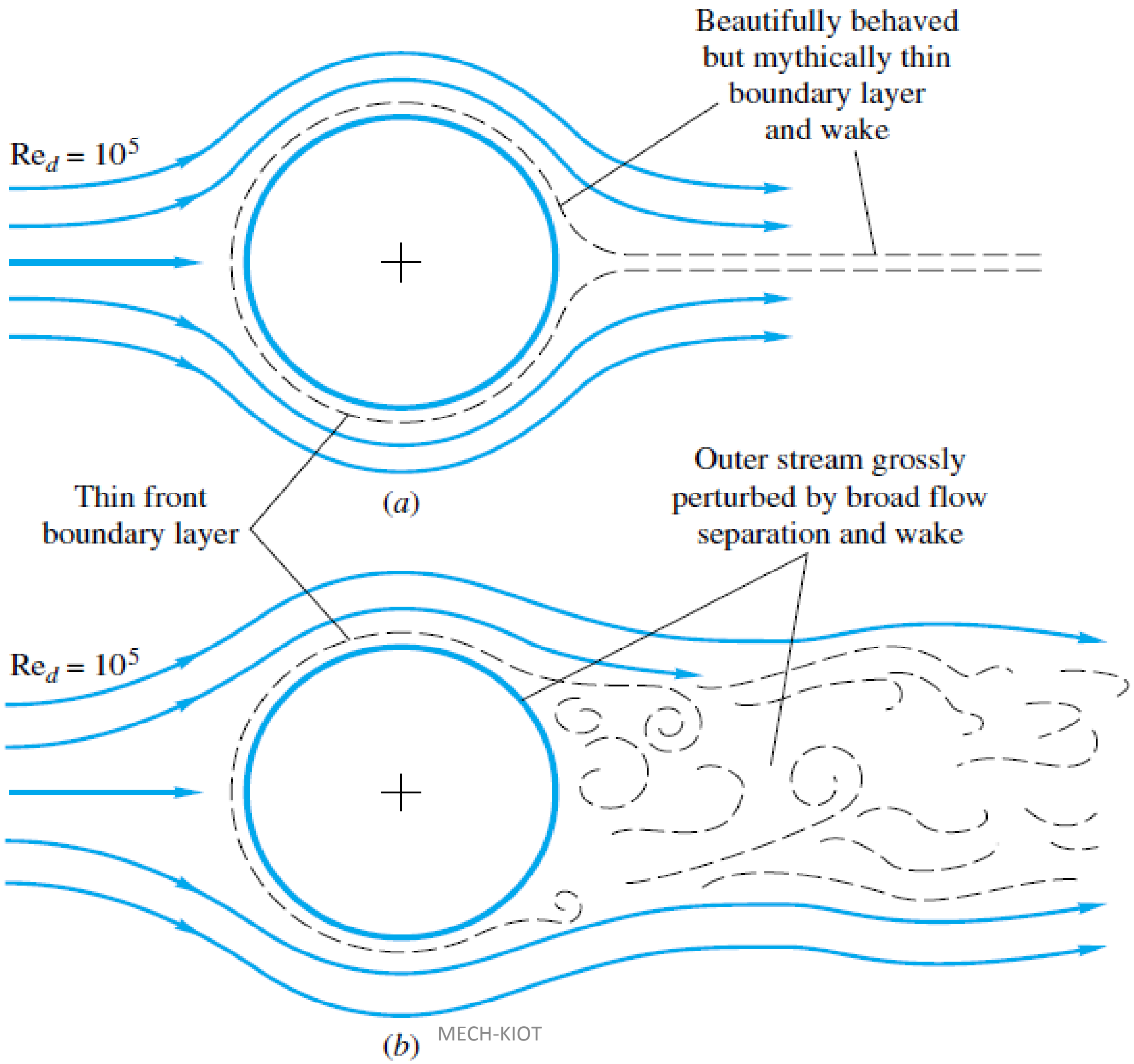


- the transition into a turbulent boundary layer

Boundary Layer Region through a Pipe



Boundary Layer Region over a Sphere



Boundary layer

Boundary layer comprises of two regions

Region I

The fluid exerts shear stress on the boundary exerts equal and opposite force on the fluid known as shear resistance. A thin layer adjoining the boundary is called boundary layer where viscous shear takes place.

Region II

The region outside the boundary layer where the flow behaviour is quite like that of an ideal fluid when the potential flow theory is applicable.

Terms associated with boundary layer

- Edge of the plate facing the direction of the flow is called leading edge

- Rear edge is called trailing edge.

- Near the leading edge of the plate boundary layer is laminar and velocity distribution is parabolic.

- Thickness of the boundary layer is increase from the leading edge as more and more fluid is slowed down by the viscous boundary, becomes unstable and breaks into turbulent boundary layer over a transition region.

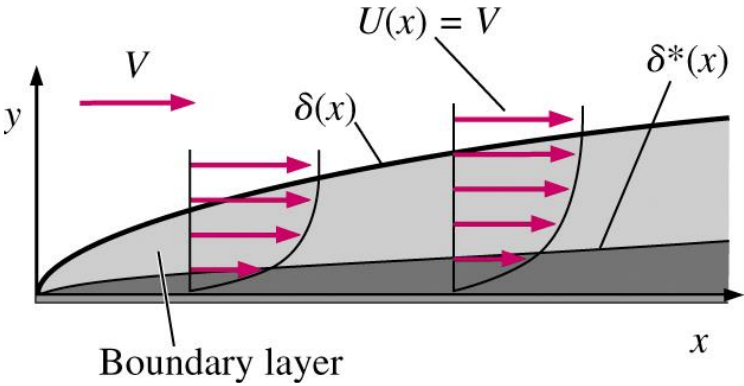
Characteristics of boundary layer

- Thickness of Boundary layer(δ) is arbitrarily defined as the distance from the boundary in which the velocity reaches 99% of velocity of the stream ($u=0.99U$)
- Definition above gives an approximate value of the B.L.T and hence it is generally termed as Nominal thickness of the Boundary layer.
- δ increases as the distance from leading edge x increases
- δ decreases as U increases
- δ increases as kinematic viscosity increases
- When U increases in the downward direction ,boundary layer growth is reduced

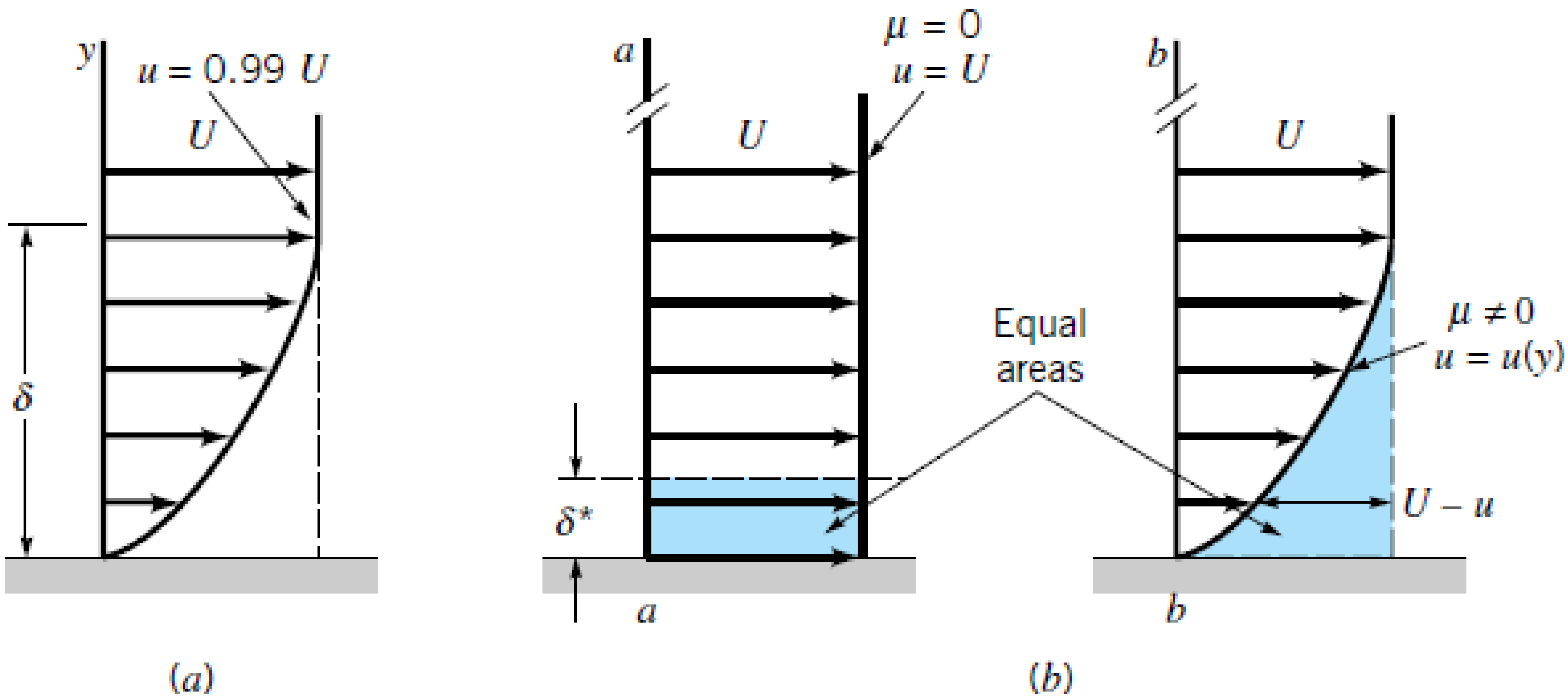
- When U decreases in the downward direction, flow near the boundary is further retarded, boundary layer growth is faster and susceptible to separation.
- The various characteristics of the boundary layer on the flat plate are governed by inertial and viscous forces.
- If $Re < 5 \times 10^5$ boundary layer is laminar (velocity distribution is parabolic)
- If $Re > 5 \times 10^5$ boundary layer is turbulent on that portion (velocity distribution follows log law or a power law)

Displacement Thickness

- Displacement thickness δ* is the imaginary increase in thickness of the wall (or body), as seen by the outer flow, and is due to the effect of a growing BL.
- Expression for δ* is based upon control volume analysis of conservation of mass



$$\delta^* = \int_0^{\infty}\left(1 - \frac{u}{U}\right)dy$$



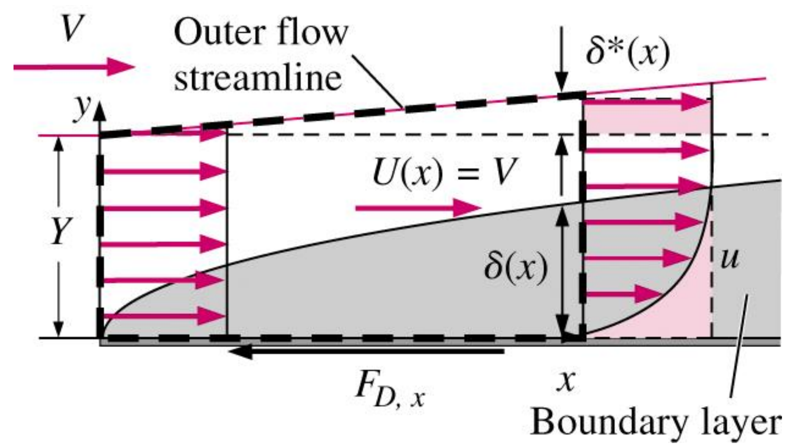
The displacement thickness represents the amount that the thickness of the body must be increased so that the fictitious uniform inviscid flow has the same mass flow rate properties as the actual viscous flow.

$$\delta^* b U = \int_0^\infty (U - u) b \, dy$$

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy$$

Momentum Thickness

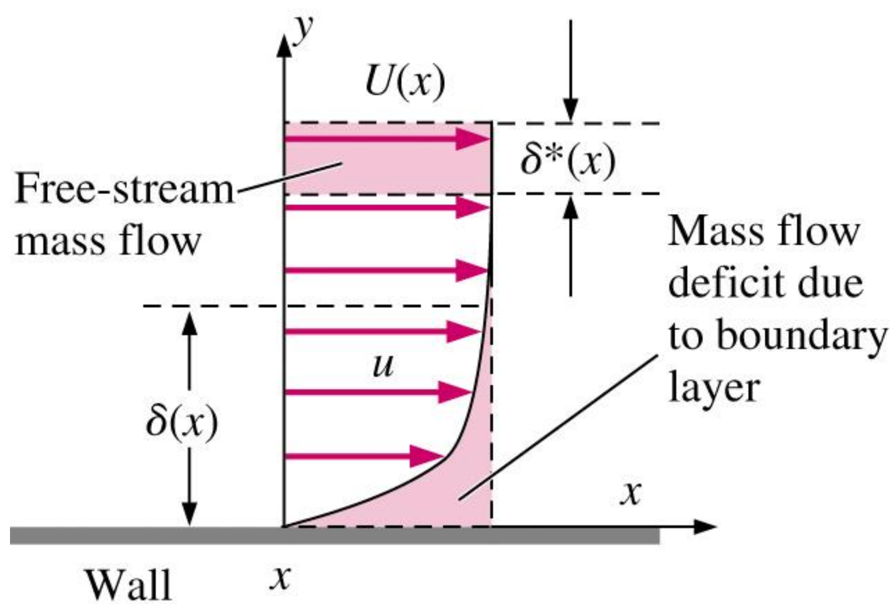
- Momentum thickness θ is another measure of boundary layer thickness.
- Defined as the loss of momentum flux per unit width divided by ρU^2 due to the presence of the growing BL.
- Derived using CV analysis.



$$\int \rho u(U - u)\, dA = \rho b \int_0^{\infty} u(U - u)\, dy$$

$$\rho b U^2 \Theta = \rho b \int_0^{\infty} u(U - u)\, dy$$

$$\Theta = \int_0^{\infty} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$



Problem

If the velocity profile in a laminar boundary layer is approximated by a parabolic profile

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

Where u is the velocity at y and $u \rightarrow U$ as $y \rightarrow \delta$. Calculate the displacement thickness and the momentum thickness

Laminar & Turbulent Boundary Layer

TABLE 10–4

Summary of expressions for laminar and turbulent boundary layers on a smooth flat plate aligned parallel to a uniform stream*

Property	Laminar	(a) Turbulent(†)	(b) Turbulent(‡)
Boundary layer thickness	$\frac{\delta}{x} = \frac{4.91}{\sqrt{\mathrm{Re}_x}}$	$\frac{\delta}{x} \cong \frac{0.16}{(\mathrm{Re}_x)^{1/7}}$	$\frac{\delta}{x} \cong \frac{0.38}{(\mathrm{Re}_x)^{1/5}}$
Displacement thickness	$\frac{\delta^*}{x} = \frac{1.72}{\sqrt{\mathrm{Re}_x}}$	$\frac{\delta^*}{x} \cong \frac{0.020}{(\mathrm{Re}_x)^{1/7}}$	$\frac{\delta^*}{x} \cong \frac{0.048}{(\mathrm{Re}_x)^{1/5}}$
Momentum thickness	$\frac{\theta}{x} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$	$\frac{\theta}{x} \cong \frac{0.016}{(\mathrm{Re}_x)^{1/7}}$	$\frac{\theta}{x} \cong \frac{0.037}{(\mathrm{Re}_x)^{1/5}}$
Local skin friction coefficient	$C_{f,x} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$	$C_{f,x} \cong \frac{0.027}{(\mathrm{Re}_x)^{1/7}}$	$C_{f,x} \cong \frac{0.059}{(\mathrm{Re}_x)^{1/5}}$

* Laminar values are exact and are listed to three significant digits, but turbulent values are listed to only two significant digits due to the large uncertainty affiliated with all turbulent flow fields.

† Obtained from one-seventh-power law.

‡ Obtained from one-seventh-power law combined with empirical data for turbulent flow through smooth pipes.

Navier Stoke Equation
Cartesian Coordinates

$$\rho\overbrace{\Big(\underbrace{\frac{\partial \mathbf{v}}{\partial t}}_{\text{Unsteady acceleration}} + \underbrace{\mathbf{v}\cdot\nabla\mathbf{v}}_{\text{Convective acceleration}}\Big)}^{\text{Inertia (per volume)}} = \overbrace{\underbrace{-\nabla p}_{\text{Pressure gradient}} + \underbrace{\mu\nabla^2\mathbf{v}}_{\text{Viscosity}}}^{\text{Divergence of stress}} + \underbrace{\mathbf{f}}_{\text{Other body forces}}.$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \rho g_x$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) + \rho g_y$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) + \rho g_z$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Thank You